THE ORBIT OF MARS
Astronomy 121
Version 2.6 – January, 1999

Instructions: Please fill in the following forms and data sheets, and hand this in as your lab. Any additional sheets you provide should be securely attached. If you have any questions please ask the teaching assistant on duty.

Name: ____

ID Number: ____

Date and Time Completed: ____________________________

Instructor: ____

Grade and Comments: ________________________________

Please sign the following special pledge:

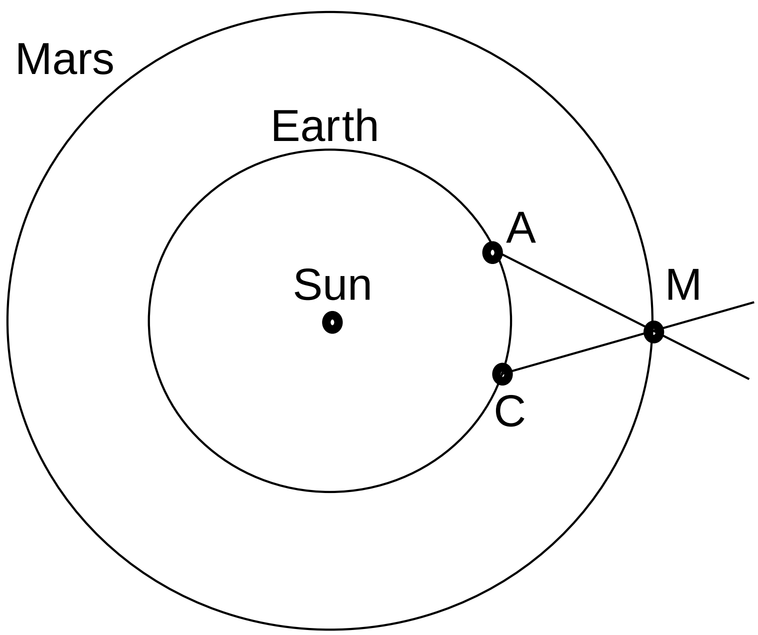
On my honor as a student, the work I am submitting here is entirely my own. I have not collaborated with or received help in any way from any other person on this laboratory (except those officially designated by the Astronomy Department). I understand that collaboration in any form would be a serious honor violation, would be considered reprehensible by the academic standards in force in this course, and will immediately be brought to the attention of the Honor Committee.

(signature)______________________ (date)______________________

NAME: ________________ ID Number: ________________ Grade: ________________

THE ORBIT OF MARS

The start of modern physics was Kepler's discovery that the planetary orbits were ellipses, not circles. He first proved this for the orbit of Mars, working from the extensive and very precise observations compiled by Tycho. His method was basically that of triangulation. He sifted the observations for times when Mars was in the same point in its orbit. He could then use observations made when Earth was at **different** points in its orbit to triangulate the distance to Mars. See the diagram below:



In the first observation, Mars appears in the direction CM as seen from Earth. In the second, in direction AM. Note that because Mars is, of course, never stationary, these two observations must have been made at least one Mars year apart. We use Kepler's method here to estimate the size of Mars' orbit.

Equipment: You will need a large piece of paper ($15'' \times 15''$), a ruler, a protractor, a compass, and a hand calculator.

You may do this laboratory at home if you wish. However, this is to be an **independent** effort, and your pledge will signify no collaboration with other people.

It may help to review the properties of an ellipse, in particular the definition of the semi-major axis, a, and the eccentricity, e. We will not actually draw an ellipse in this lab. Rather we will approximate an ellipse by an offset circle. The offset circle will be a good approximation to Mars' orbit only near the major axis of the true elliptical path.

Procedure

- Write your name on the large sheet of paper.

  Find the center of your paper (the Sun) and draw a straight line from the center to the right side of the paper. This represents the basic "zero direction" in space, the vernal equinox. Label this direction. Draw a circle of radius 10 cm to represent the Earth's orbit. Mark the position of the Earth for each date in the following table, and label the date. This is found from the **heliocentric longitude** measured with a protractor from the vernal equinox direction that you already labelled.

  Next, rather than measuring with the sun at the center, use each different position of the Earth as the center of your measurement, and 0° in a direction **parallel** to the vernal equinox, mark off the directions for the geocentric longitude of Mars. Draw a line between the position of Earth at each date and the corresponding geocentric longitude of Mars which you have just determined. You will find that the lines fall into two groups, each of which converges on a location for Mars. These are the actual observations used by Kepler and were deliberately chosen to represent perihelion and aphelion. Find the midpoint between these two positions and draw a circle centered on the midpoint to approximate the orbit of Mars. On the diagram, label the following: The center of Mars' orbit, the Sun, the major axis of Mars, the minor axis of Mars, and the locations of Mars' perihelion and aphelion.

Date	Heliocentric Longitude of Earth	Geocentric Longitude of Mars
1585 Feb 17	159° 23′	135° 12′
1587 Jan 05	115° 21′	182° 08′
1600 Mar 06	176° 32′	119° 18′
1591 Sep 19	005° 47′	284° 18′
1593 Aug 06	323° 26′	346° 56′

- The observations we chose here represent opposite ends of the major axis (i.e. the perihelion and aphelion) of Mars' orbit. Draw in the major axis. Measure the semi-major axis, a, and express it in Astronomical Units (i.e. the radius of the Earth's orbit):

  a= ______________________________ A.U.

- The offset between the center of the circle and the Sun is the distance ae –that is, the semi-major axis times the eccentricity. Measure this distance and calculate the eccentricity of Mars' orbit:

  e= ______________________________

- You were not expected to get exactly correct answers from this exercise. How much error would you estimate is present in your method? That is, if you did the problem over again from scratch, how much would you estimate your second answer for a would differ from your first? Express as a percent: ______________%.
  The correct values are $a = 1.52$ AU and $e = 0.093$. Is your estimated error consistent with the actual difference between the true value and your value for a? ____(yes,no).

- The true orbit is an ellipse. The offset circle we have used is a good approximation for points near the major axis but is not expected to work well near the minor axis (i.e. 90° from the major axis). The semi-minor axis, b, can be estimated from the properties of an ellipse as follows:

$$b = a\sqrt{1-e^2}.$$

  So, b= ________________________________AU.
  To determine whether the difference between the offset circle approximation and the ellipse could be observed, do the following: First, calculate the difference between a and b, which will be the difference between the true and approximate orbit near the minor axis: a–b = ____________________________AU.
  Now return to your figure. Pick any one of the Earth positions used in the table above and measure the distance D from that position to the point where Mars is on the minor axis of its orbit. Now use the small angle approximation:

$$\theta = \text{(a–b)/D},$$

  where (a–b) is the deviation to estimate the angular discrepancy observable from Earth. In this formula, the angle is expressed in *radians*; you will have to convert it to degrees. To do so, multiply by 57.3. θ = ____________________degrees.
  Would this discrepancy be visible to naked eye instruments such as Tycho's? (Assume he could determine positions accurate to 0.03°): ________________(yes,no).

- Use Kepler's Third Law (see the textbook) to estimate the period of Mars' revolution about the Sun: Period = ________________________years.
  The true period is 1.88 years. What % error did you make? ____________%.

- Mars and Venus both undergo large variations of brightness as a result of their varying distance from Earth. The size of these variations was, in fact, one of the clues which led Copernicus to his heliocentric theory. Here we will estimate the variations in apparent brightness expected for Mars. The magnitude of an object is a function of the **flux** received from it; the flux in turn varies as the **inverse square** of the distance of the object. From the definitions of the magnitude system, it turns out that

$$m_2 - m_1 = 5\log\left(\frac{d_2}{d_1}\right)$$

  where m represents the apparent magnitude and d the distance to an object. 1 and 2 refer to different distances.

Using your diagram for the orbits of Earth and Mars, estimate the minimum and maximum distance **possible** between the two planets.

minimum = ____AU.

maximum = ____AU.

How many times more flux will we receive from Mars at minimum distance compared to maximum? The flux ratio scales as $(d_{max} / d_{min})^2$. __________times more flux.

What will be the corresponding difference in apparent magnitude between minimum and maximum distance? __magnitudes different. If Mars is –2 magnitude at brightest, what will its magnitude be at its maximum distance? ____________magnitudes.

- If there are 2 stars brighter than m=0, 40 brighter than m=2, 500 brighter than m=4, and 5000 brighter than m=6, would you say that Mars will be conspicuous at its brightest? ______________(yes,no). At its faintest? ______________(yes,no).

Attach your worksheet to this writeup