EXPOSURE TIME ESTIMATION

An essential part of planning any observation is to estimate the total exposure time needed to satisfy your scientific goal. General considerations are described in Lecture 7. A number of on-line "Exposure Time Calculators" (e.g. at STScI) provide examples of sophisticated estimators. Here, we discuss the basics of this process.

From Lecture 7.F, we can re-write a general expression for the signal-to-noise ratio for digital array photometry as follows:

$$SNR = \frac{N\dot{s}\,t}{\sqrt{N\dot{s}\,t + N(1+\frac{N}{M})\left(\dot{b}t + \dot{d}t + n_{\mathrm{RON}}^2\right)}} \qquad (1)$$

Here, $\dot{s}$ and $\dot{b}$ are the <u>mean rates</u> at which source photons and sky background photons, respectively, are detected <u>per pixel</u>. t is the total integration time. $\dot{d}$ is the dark count rate per pixel, and n_{RON} is the readout noise per pixel (quoted in numbers of equivalent electrons of noise). We assume the source was measured with a virtual aperture containing N pixels and the background with an aperture of M pixels, where $M \geq N$.

This expression is usable for either a compact (e.g. point-like) source or an extended source. In the case of a compact source which is completely contained within the N pixel source aperture, $\dot{s} = \dot{S}/N$, where $\dot{S}$ is the total source photon detection rate.

The next question is how to estimate the photon rates appearing in this expression, starting from an estimate of the flux of the source at the top of Earth's atmosphere. It is straightforward to do this.

The flux of detected photons (counts per second per cm^2) at the detector for a given source will be:

$$\Phi_i = \frac{\frac{\pi}{4}D_e^2}{\frac{\pi}{4}(\beta L)^2} \int e^{-k(\lambda)\,\sec Z} T_i(\lambda) \frac{F_\lambda(\lambda)}{h\nu} d\lambda \qquad (2)$$

Here:

D_e **is the <u>effective</u> diameter of the telescope after correction for central obscuration. (The correction can be significant, depending on the optical design.)**

β **is the angular diameter of the source (radians).** $\beta = 4.85 \times 10^{-6} \beta_s$, **where β_s is the angular diameter in arcseconds.**

L **is the effective focal length of the optical system feeding the detector.**

k **is the "extinction coefficient" for absorption in Earth's atmosphere. (This is small, $\lesssim 0.2$ except in the near-UV and can usually be ignored for ET estimates.) k is smaller for higher altitude observatories. An extinction curve for Mauna Kea is shown in Lecture 4.**

$\sec Z$ **is the secant of the "zenith distance" angle. This quantity is also known as the "air mass," since it gives the total atmospheric path length toward the source in terms of the vertical thickness of the atmosphere.**

T_i **is the net system optical efficiency. This includes the fractional transmission of the optical system, including the telescope and all other optical elements preceding the detector, and the detector quantum efficiency.**

$F_\lambda(\lambda)$ **is the spectral flux density of the source, in energy units per unit wavelength, at the top of Earth's atmosphere. For an <u>extended</u> source, enter $F'_\lambda(\lambda)$, the mean spectral flux density per square arcsecond, and set $\beta_s = 1$.**

Estimating $F_\lambda(\lambda)$:

If you can estimate m_λ, the monochromatic magnitude of the source, then:

$$F_\lambda(\lambda) = 3.63 \times 10^{-9} \, 10.0^{-0.4\,m_\lambda(\lambda)} \ \mathrm{erg \ s^{-1} \ cm^{-2} \ \mathring{A}^{-1}}$$

for any wavelength.

More commonly, you have an estimate of the source brightness in a particular band of a broad-band magnitude system. If the magnitude in band i is m_i, then

$$< F_\lambda(\lambda) >= F_{0,i} \, 10.0^{-0.4\,m_i} \ \mathrm{erg \ s^{-1} \ cm^{-2} \ \mathring{A}^{-1}}$$

where $F_{0,i}$ is the flux zeropoint for band i. The best compilation of such zeropoints is by Bessell et al. (A&A, 333, 231, 1998; Table A2). See Lecture 13 for more details. The estimate here is, of course, for the <u>mean flux</u> within the band.

For an extended source, you would substitute $\mu_\lambda(\lambda)$, in standard units per square arcsecond, for m in the expressions above. See Lecture 2 for the definition of μ.

Estimating $T(\lambda)$:

Accurate determinations of the total optical efficiency of a system can only be made after observing calibrator sources. However, the following rules of thumb serve adequately in the absence of more detailed information.

$T(\lambda)$ is the product of the transmissions of each optical element in the system and the detector quantum efficiency.

$$T(\lambda) \approx r_m^{n_m}(\lambda) \; r_l^{2\,n_l}(\lambda) \; R_f(\lambda) \; Q(\lambda)$$

Here:

r_m is the reflectivity of a mirror and n_m is the number of mirrors in the optical path. For fresh aluminum coatings on mirrors $r_m \sim 0.88$ in the optical bands. Silver has better IR reflectivity but poorer blue reflectivity. In the UV, special coatings such as MgF or LiF are used.

r_l is the transmissivity of a single glass-air interface and n_l is the number of refractive elements in the optical path. For normal glasses in the optical band, $r_l \sim 0.95$. Should include dewar windows, etc., in this factor.

$R_f(\lambda)$ is the filter (or grating, prism) transmission curve. Typical glass filters have $< R > \sim 0.40 - 0.90$ (including the effects of the glass/air interface). Inteference filters have generally lower values. Since it is usually easy to look up filter properties, and filter responses vary dramatically, this is one term that you should try to estimate accurately.

$Q(\lambda)$ is the quantum efficiency of the detector. Again, this can usually be looked up.

This expression assumes no other light losses (e.g. due to vignetting of the light beam).

Note that this long string of factors will normally result is only <u>modest</u> overall throughput. For instance, in a standard Cassegrain telescope with 2 mirrors, 2 refractive elements, a filter with 75% transmission, and a CCD with 50% QE, the net throughput is only 0.24. 75% of the photons incident on the primary mirror have been lost!

Metal Reflectivities
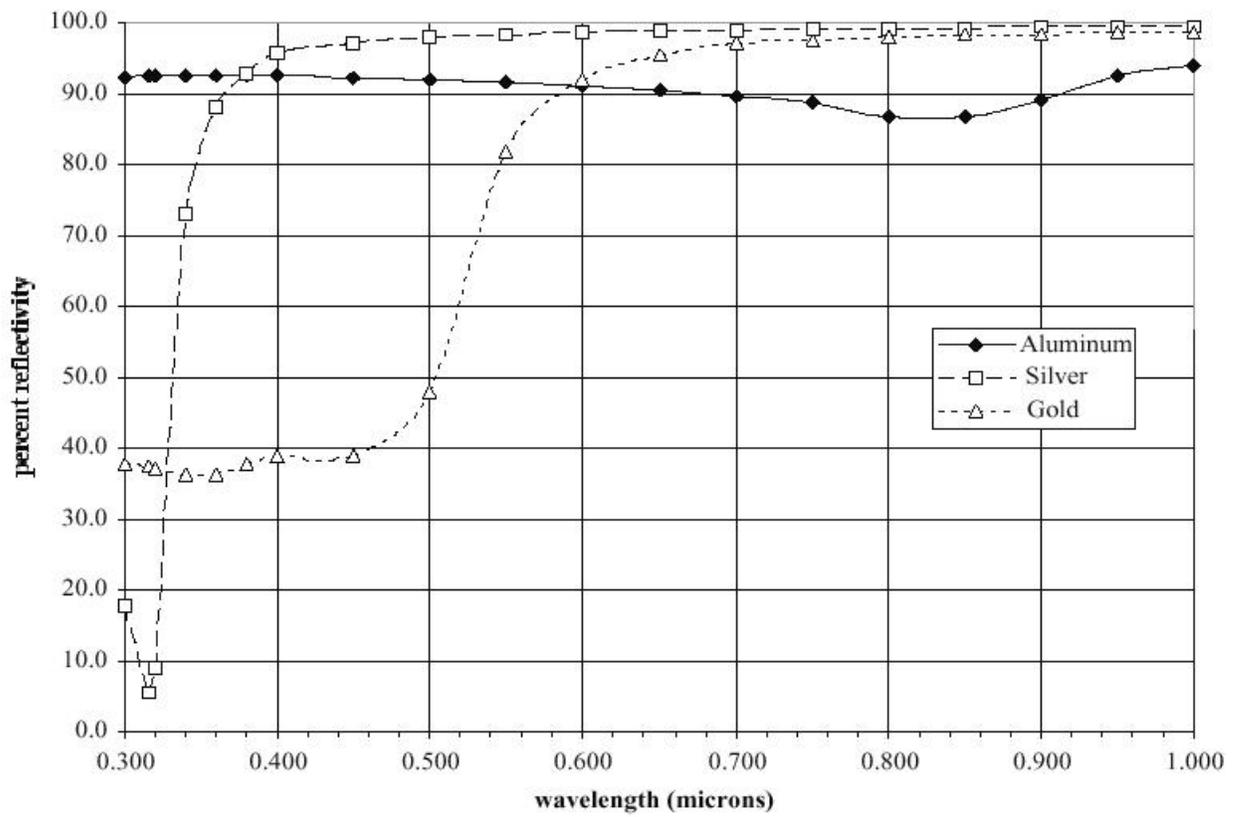
Reflectivities of common large mirror coatings

Approximating the Flux Integral (equation 2):

If we substitute mean values in (2) above, we get:

$$\Phi_i \approx \frac{D_e^2}{(\beta L)^2} < T_i > \frac{< F_{\lambda,i} >}{h\nu} \Delta\lambda$$

where $\Delta\lambda$ is the bandwidth of the filter. This is obviously a critical term and should be carefully determined. A standard approach is to compute $\Delta\lambda = \int \frac{R_f(\lambda)}{R_0} d\lambda$, where R_0 is the peak response of the filter.

Values for the standard UBV filters are given below, and plots of the UBVRI responses are given on the next page.

Band	$< \lambda >$ (Å)	$\Delta\lambda$ (Å)
U	3600	560
B	4400	990
V	5500	880

If the bandwidth is particularly large, the source energy distribution changes significantly across the band, or $T(\lambda)$ is not well represented by a top-hat function, then good results may require that the actual integral be evaluated.
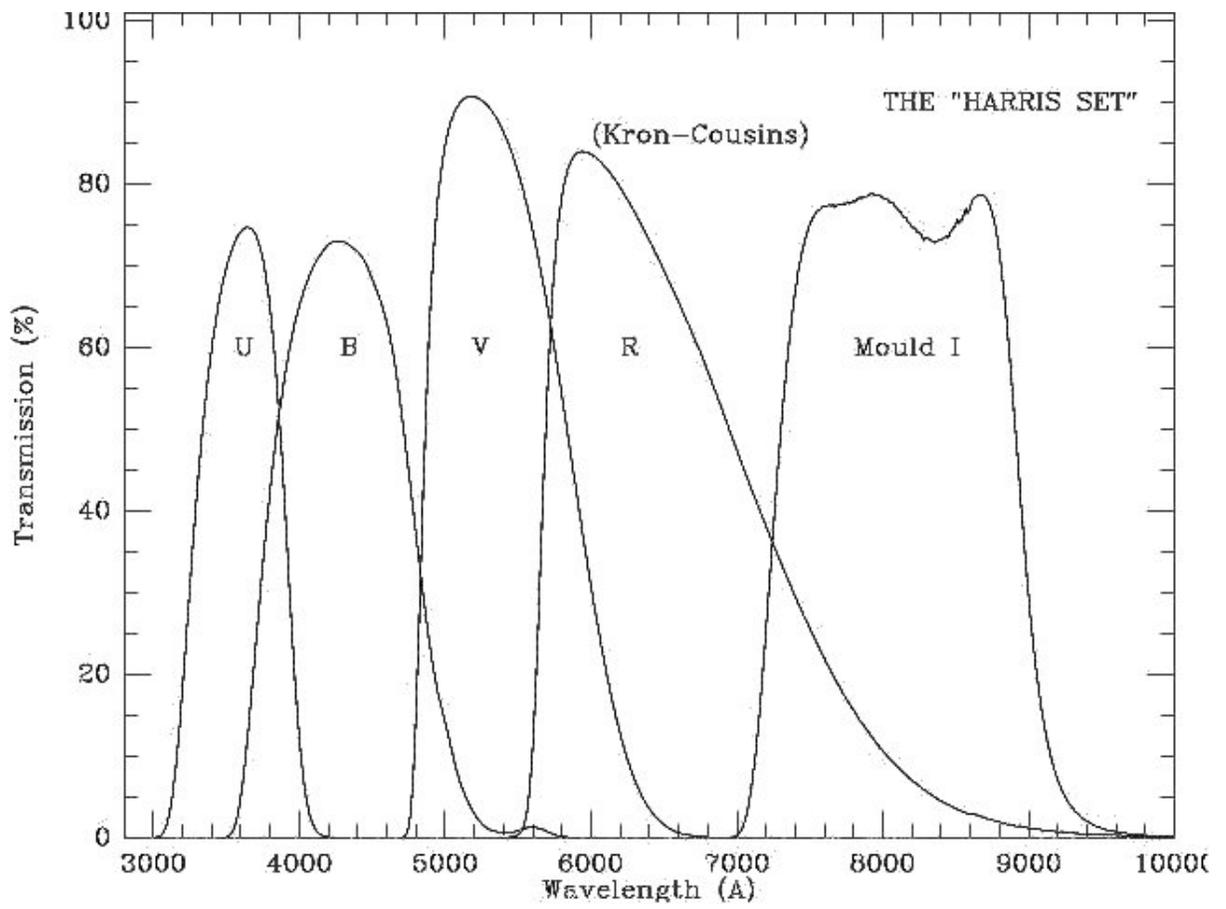
Standard UBVRI broad-band filter response curves (KPNO)

Predicting Signal-to-Noise:

To complete evaluation of equation (1):

> Evaluate $\Phi_{s,i}$ for the source by entering $< F_{\lambda,i} >$ for a compact source and the appropriate β_s; or entering $< F'_{\lambda,i} >$ for an extended source with $\beta_s = 1$. For a point source, β_s is the FWHM of the point spread function of the telescope+atmosphere in arcseconds.

> Then the term $\dot{s}$ in equation (1) is

$$\dot{s} = \Phi_{s,i}\, y_p^2$$

> where y_p is the size of one pixel in cm (assuming square pixels).

> This expression applies for those pixels which are within the projected image of the source. If the source is compact, such that its light is completely contained within the source measuring aperture of N pixels, then

$$N\dot{s} = \Phi_{s,i}\, \frac{\pi}{4}(\beta L)^2$$

> If the source is not contained within the N pixels but is also not well approximated by a uniform surface brightness, then you must estimate the total flux in the source aperture using an assumed spatial profile.

> Then evaluate $\Phi_{b,i}$ for the sky background by entering the $< F'_{\lambda,b,i} >$ which is appropriate for the sky surface brightness, $\mu_{b,i}$ (typical values for dark sites are given in Lecture 7). Use $\beta_s = 1$. Then:

$$\dot{b} = \Phi_{b,i}\, y_p^2$$

> Obtain values for $\dot{d}$ and n_{RON} from published specifications for the detector you are using.

> Substitute all of the above into equation (1).

> It is easiest to solve equation (1) numerically for various sets of assumptions. The most common approaches are: (i) to solve for SNR given source/background brightnesses and exposure time t; (ii) to solve for t given source/background brightnesses and SNR; and (iii) to solve for the source brightness given SNR, t, and the background.

> Remember that a <u>threshold</u> detection requires $SNR \sim 5 - 10$.

> For a plot of a type (ii) evaluation for the 40-in CCD system in the V-band, see the next page.

40-in Integration Times, Point Source, Sky = 21
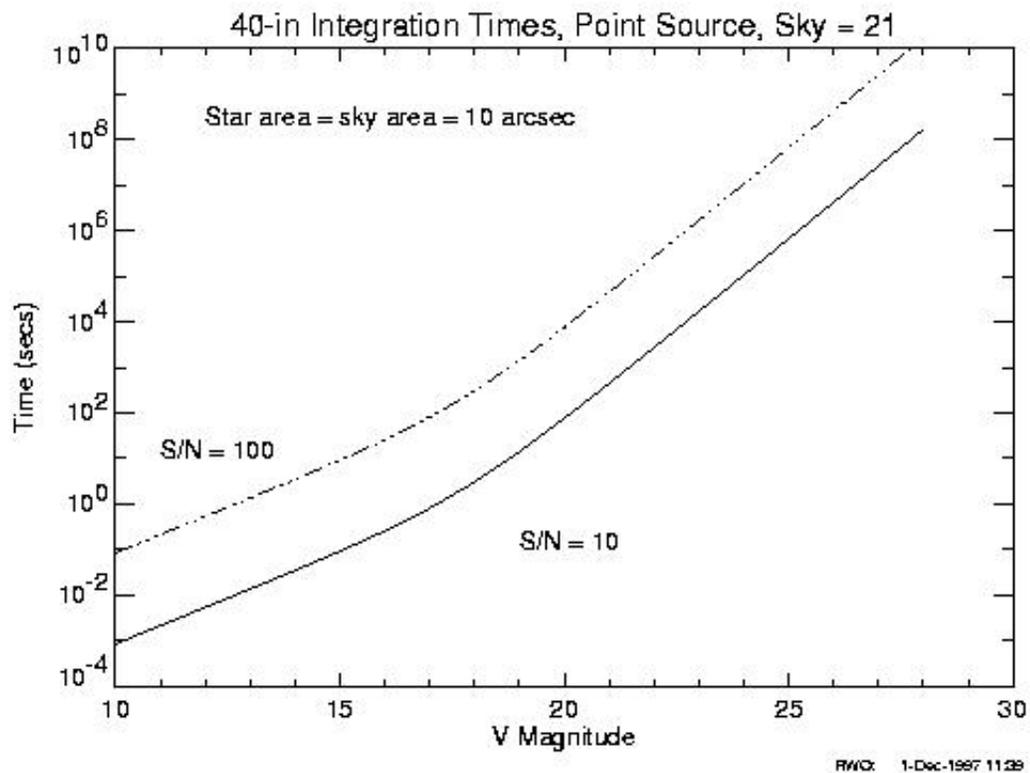
Example estimate of integration times for the 40-in CCD imaging system including the effects of source noise, sky background, and readout noise.

Note the change of slope at $V \sim 18$, which is the transition between sky background noise dominance (for fainter sources) and source photon noise dominance (for brighter sources). Because this is a log-log diagram, such small changes in slope have large impacts in practice.

Limiting Cases (Point Sources):

The dependence of the results on the parameters of the telescope plus instrument system are best illustrated in limiting cases.

(1) Source limited: $\dot{s} >> \dot{b}$

$$SNR \sim D_e \sqrt{F_\lambda T \Delta\lambda t}$$

(2) Sky background limited: $\dot{b} >> \dot{s}$ (assuming $M >> N$)

$$SNR \sim D_e F_\lambda \sqrt{\frac{T \Delta\lambda t}{N F'_{\lambda,b,i}}}$$

Note that in both cases,

$$t \sim \frac{(SNR)^2}{D_e^2 T \Delta\lambda}$$

This implies, as noted in Lecture 7, that it is expensive in observing time to increase SNR. There is also an important advantage for larger telescopes. However, the dependence on $F_{\lambda,i}$ is very different in the two cases. In the sky limited case, $t \sim F_\lambda^{-2}$, meaning that faint sources become very difficult to detect.

(3) A final interesting case is the sky background limit for a point source with a <u>diffraction limited</u> telescope, for which $\beta \sim 2.4\,\lambda/D_e$. Here, we assume the source measuring aperture is decreased in proportion to β. Then

$$t \sim \frac{\lambda^2}{D_e^4} (SNR)^2$$

Here, there is an enormous advantage for larger telescopes. This case applies in practice to optical-IR space telescopes (HST, JWST) or to large ground-based telescopes operating in the IR region using "adaptive optics" to eliminate the image smearing from the Earth's atmosphere.

One caveat, among others: the IR sky background from the Earth's atmosphere is strongly variable and is not well approximated by Poissonian statistics.

Final Remarks:

On-line Exposure Time Calculators are for specific instruments. They include all the relevant information on T, R_f, Q, and so forth, and produce real integrals across the bandwidth. They generally accept a number of different ways of specifying or estimating source brightness. They usually include a high-resolution version of the background sky spectrum, so the effects of strong sky emission lines are properly estimated.

Finally, remember that equation (1) is a <u>prediction</u> of SNR for a specific set of assumptions about the measuring process.

The best way to judge the <u>true SNR</u> of an observation is to analyze the <u>scatter in repeated measures</u>.

Therefore, you should always break an observation up into <u>multiple parts</u> (assuming it is not dark or readout-noise limited) in order to assess statistical scatter as well as to reduce the effects of cosmic rays, flat field uncertainties, etc.