SIMPLE RADIATIVE TRANSFER

The theory of radiative transfer provides the means for determining the emergent EM spectrum of a cosmic source and also for describing the effects of media through which the radiation passes on its way to final detection.

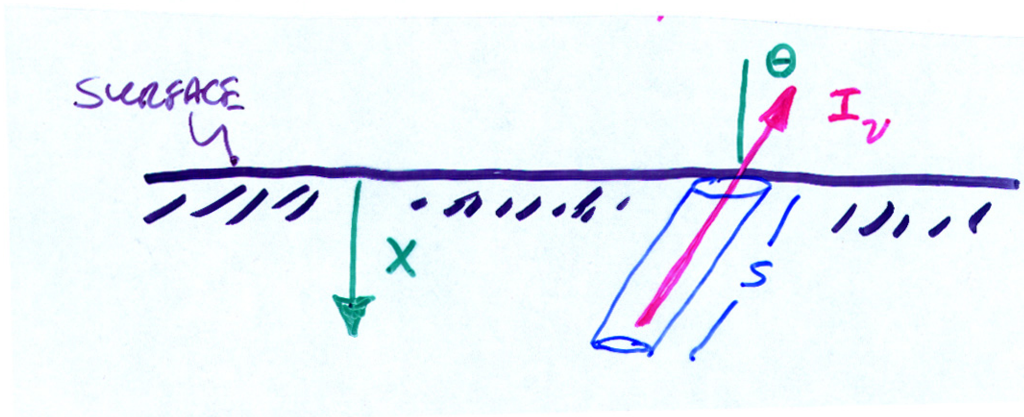
References:

- **LLM, Section 5.1**
- **Gray (1992) "Observation & Analysis of Stellar Photospheres"**
- **Kourganoff (1963) "Basic Methods in Transfer Problems"**
- **Mihalas (1978) "Stellar Atmospheres"**
- **Shu (1991) "The Physics of Astrophysics: Vol. 1 Radiation"**
- **Zeldovich & Raizer (1966) "Physics of Shock Waves & High Temperature Hydrodynamic Phenomena"**

Only simple discussion here, for plane-parallel, 2D case

Details: ASTR 543-4

THE EQUATION OF TRANSFER: describes the change in <u>specific intensity</u> for photons traveling a distance s in a specific direction at a given position x in a source



$$dI_\nu(x,\nu,\theta) = -\kappa(x,\nu) I_\nu \, ds + j(x,\nu,\theta)\, ds$$

The EOT contains destruction (κI) and creation (j) terms

κ**: Absorption coefficient. Units:** cm^{-1}

j**: Emission coefficient. Units:** $\mathrm{ergs\ s^{-1}\,cm^{-3}\,Hz^{-1}\,sterad^{-1}}$

Relation to quantities per particle:

$\kappa = \sum_i n_i \alpha_i(\nu)$**, where α is the radiative cross section (cm^2) per particle at ν**

$j = \sum_i n_i \epsilon_i(\nu,\theta)$**, where ϵ is the emission coefficient (intensity units) per particle at ν**

. . . and n_i is the density of particles of type i (units cm^{-3})

NB: both κ and j include effects of photon <u>scattering</u>—i.e. redirection of photons in θ

Solution of EOT

Define: $\mu = \cos\theta$, **and**

"optical depth" $\tau(\nu) = \int_0^x \kappa(x,\nu)\,dx$

(recall x measures physical depth into source)

Then rewrite EOT as

$$\mu \frac{dI_\nu}{d\tau} = I_\nu - S_\nu$$

. . . where $S_\nu \equiv \dfrac{j(\nu)}{\kappa(\nu)}$ is the "source function"

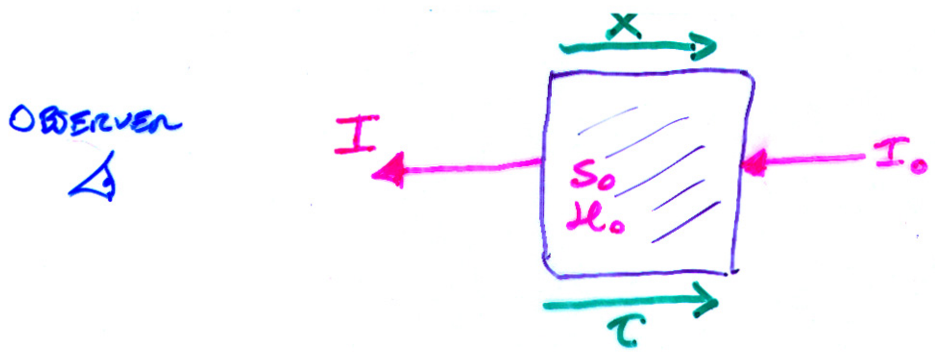
Formal solution, for I at given ν (and $\theta = 0$) at surface of plane-parallel slab with optical depth τ_1:

$$I_\nu = I_\nu(\tau_1)\,e^{-\tau_1} + \int_0^{\tau_1} S_\nu(t)\,e^{-t}dt$$

The emergent intensity is the integral of the source function at a given optical depth in the layer weighted by $e^{-\tau}$ plus the fraction of the incident intensity which escapes absorption in the source (given by $e^{-\tau_1}$).

The integral is straightforward if S is known in advance. But if the radiation field is important in determining the distribution of matter over states, then S will depend on I (over all ν), so that one must solve an integral equation. Techniques for doing this are in the references & ASTR 543-4.

Application to a Uniform "Slab" Source



- Plane-parallel, homogeneous source, intensity I_o incident on back side; no photon scattering
- For given ν, source function, emission, absorption coefficients are constant: j_o, κ_o, $S_o = j_o/\kappa_o$,
- Total physical depth $x_o \rightarrow$ optical depth $\tau_o = \kappa_o x_o$
- Then solution to EOT is

$$I = S_o(1 - e^{-\tau_o}) + I_o e^{-\tau_o}$$

- Limiting cases

  o Optically thick: $\tau_o(\nu) >> 1$: $I = S_o$

  Interpretation: see into source a distance $\sim$ one optical depth, or $x_1 \sim 1/\kappa_o \rightarrow I \sim j_o x_1$
  Incident radiation totally extinguished

  o Optically thin: $\tau_o(\nu) << 1$: $I = \tau_o S_o + I_o(1 - \tau_o)$
  $= j_o x_o + I_o(1 - \tau_o)$

  Interpretation: see all photons generated in observer direction by slab; see all but fraction τ_o of incident photons

LTE

Kirchoff's Law:

- In strict thermodynamic EQ at temperature T,

$$I_\nu = B_\nu(T) \text{ and } dI/d\tau = 0$$

- The EOT then requires that

$$I_\nu = B_\nu(T) = S_\nu = j(\nu)/\kappa(\nu)$$

- $\rightarrow$ Kirchoff's Law: $j(\nu)/\kappa(\nu) = B_\nu(T)$ in TEQ

Local Thermodynamic Equilibrium:

- If $S_\nu = j(\nu)/\kappa(\nu) \sim B_\nu(T)$ even if $I_\nu \neq B_\nu$, this is "LTE"
- Occurs where local collisions govern distribution over states and radiation is relatively weakly coupled to local matter
- A remarkable simplification, considering complexity of interactions & number of microstates affecting j and κ
- Often applies to dense astrophysical sources: e.g. stellar atmospheres.

- LTE slab: $I_\nu \sim B_\nu(T)(1 - e^{-\tau_o}) + I_0\, e^{-\tau_o}$, where T is a characteristic temperature

APPLICATION TO STELLAR PHOTOSPHERES

Combining relations above, the emergent intensity from a stellar atmosphere in LTE will be (for $\theta = 0$)

$$I_\nu(\nu) = \int_0^\infty B_\nu(\nu, T(\tau))\, e^{-\tau} d\tau$$

where τ is the optical depth at ν.

Basic computational problem: determine the $\underline{T(\tau)}$ function. This requires solving a set of simultaneous integral equations.

- "Grey" Atmosphere: $\kappa = const$, independent of ν

  τ is the same for all ν at a given physical depth x

  Solve by various approximation techniques (see references). Solution is:

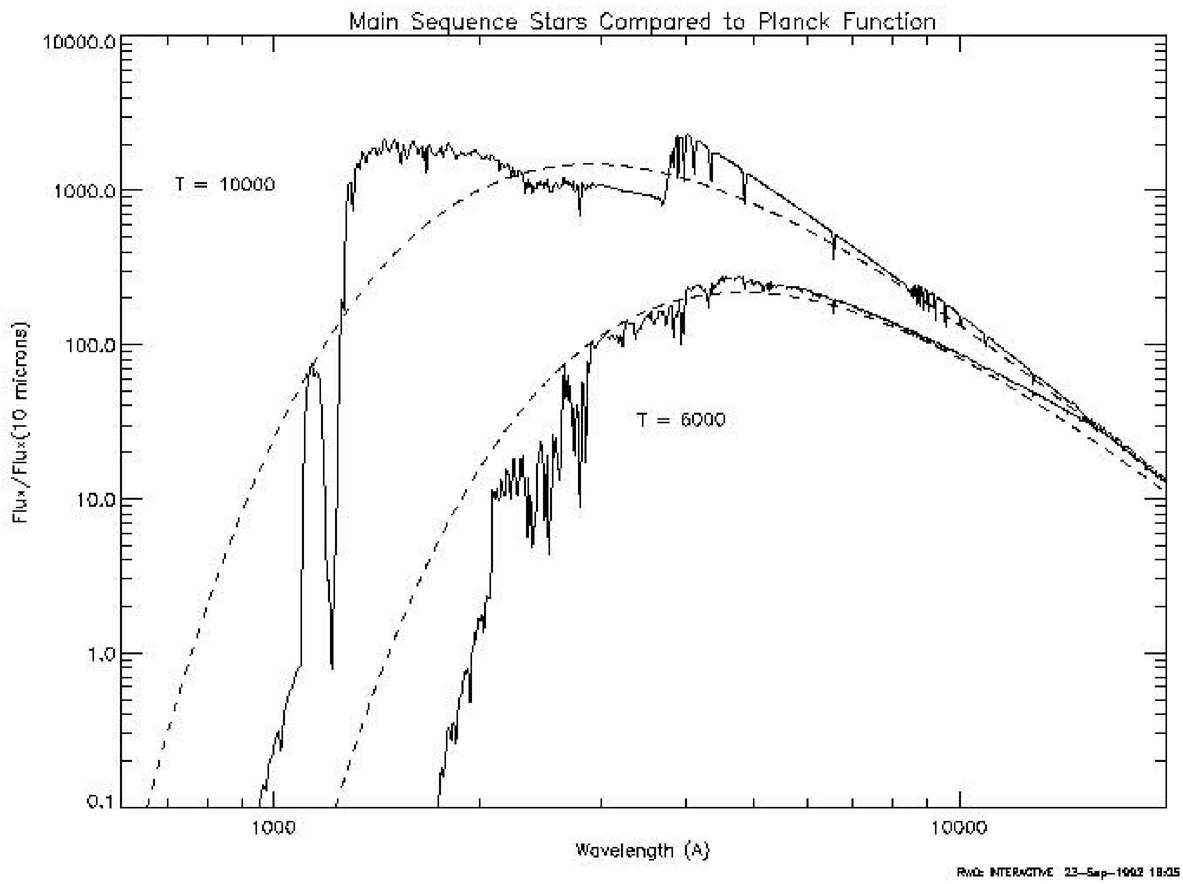
$$T^4(\tau) \sim \frac{3}{4}(\tau + \frac{2}{3})T_e^4$$

  ... where the "effective temperature" is defined by $T_e^4 \equiv F_0/\sigma_0$, and F_0 is the emergent bolometric flux at the top of the atmosphere

PHOTOSPHERES (continued)

- **The grey solution is a slowly varying function of wavelength, and $I_\nu \sim B_\nu(T_e)$.**
- **Useful approximation for various ROM applications.**
- **Departures from the grey solution occur because the real opacity can be a strong function of wavelength—e.g. in spectral "features" (absorption lines or continuum discontinuities) or in continuous opacity sources (e.g. H^- in solar-type stars).**
- **For ν's where opacity is large, radiation emerges from layers closer to surface. These have <u>lower</u> temperatures, so the SED there falls below the grey approximation.**
- **But the energy absorbed where opacity is high must emerge elsewhere in the spectrum to conserve overall energy in the radiation flow; there output exceeds the grey approximation. See plots next page.**

PHOTOSPHERES (continued)



Comparison between true emergent spectrum from stellar photosphere and Planck functions

SOME UVOIR OBSERVING APPLICATIONS

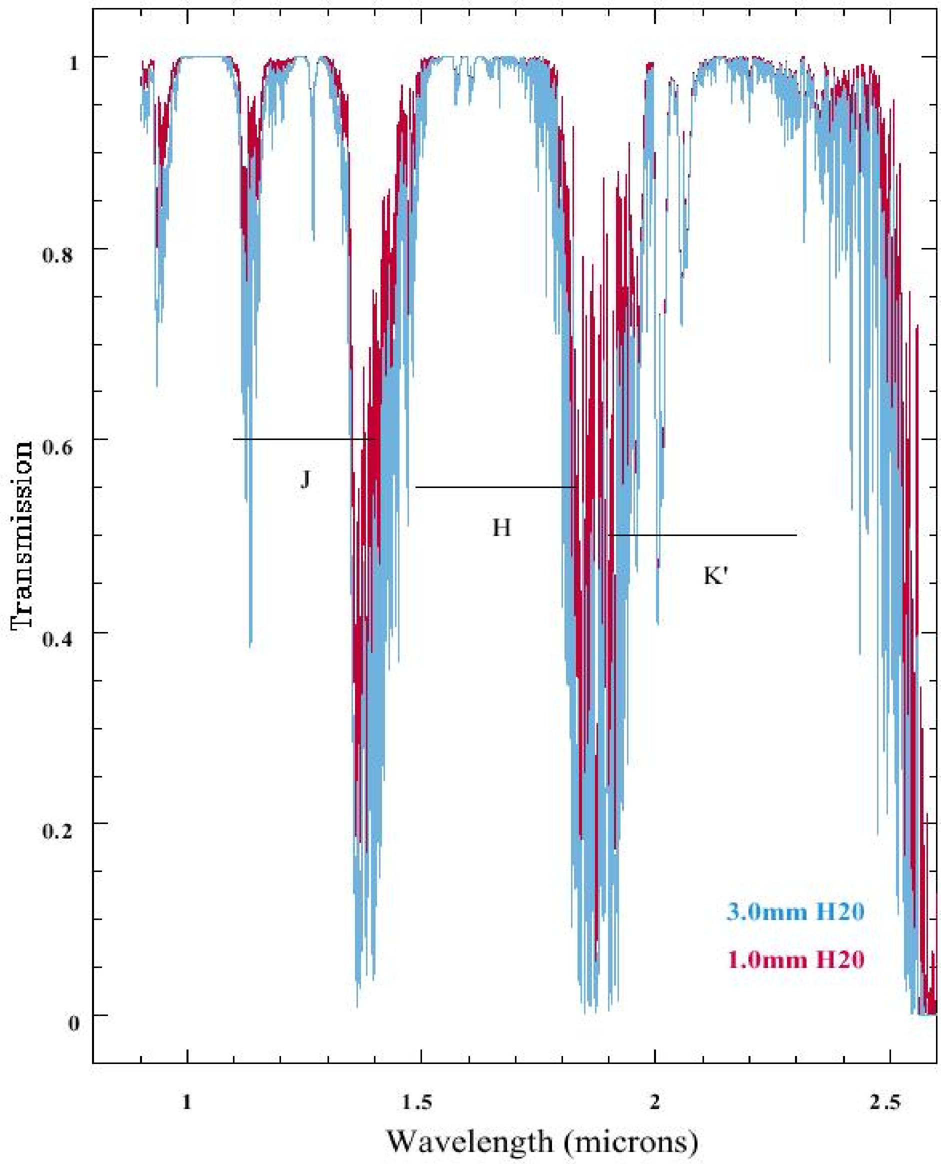
A. TRANSFER THROUGH EARTH'S ATMOSPHERE

- **Treat as LTE slab:** $I_\nu \sim B_\nu(T)(1 - e^{-\tau_o}) + I_0\, e^{-\tau_o}$ **($\theta = 0$ assumed)**
- **Molecular absorptive opacity (e.g. from H_2O) is important in the IR. Slab solution shows that absorption must be accompanied by thermal emission.**
- **Knowing $\tau(\nu)$ in a given band, can estimate effects of both absorption and emission of atmosphere by putting $T \sim 270$ K into LTE slab solution.**
- **Note that the peak of B_ν at 270 K is at $\sim 20\mu$**
- **Combined absorption and contaminating emission seriously affect observations where τ is finite and $B_\nu(T)$ is large (mostly $\lambda > 1.5\mu$)**
- **Where B_ν at 270 K is small (e.g. $\lambda < 1\mu$), the atmosphere produces "extinction" given by $e^{-\tau_o(\nu)}$ term without re-emission.**
- **So, in optical bands, $I = I_0\, e^{-\tau_o(\nu)}$, and the extinction in magnitudes is**

$$\Delta m \equiv -2.5 \log_{10}(I/I_0) = 2.5\, \tau_o(\nu) \log_{10} e = 1.086\, \tau_o(\nu)$$

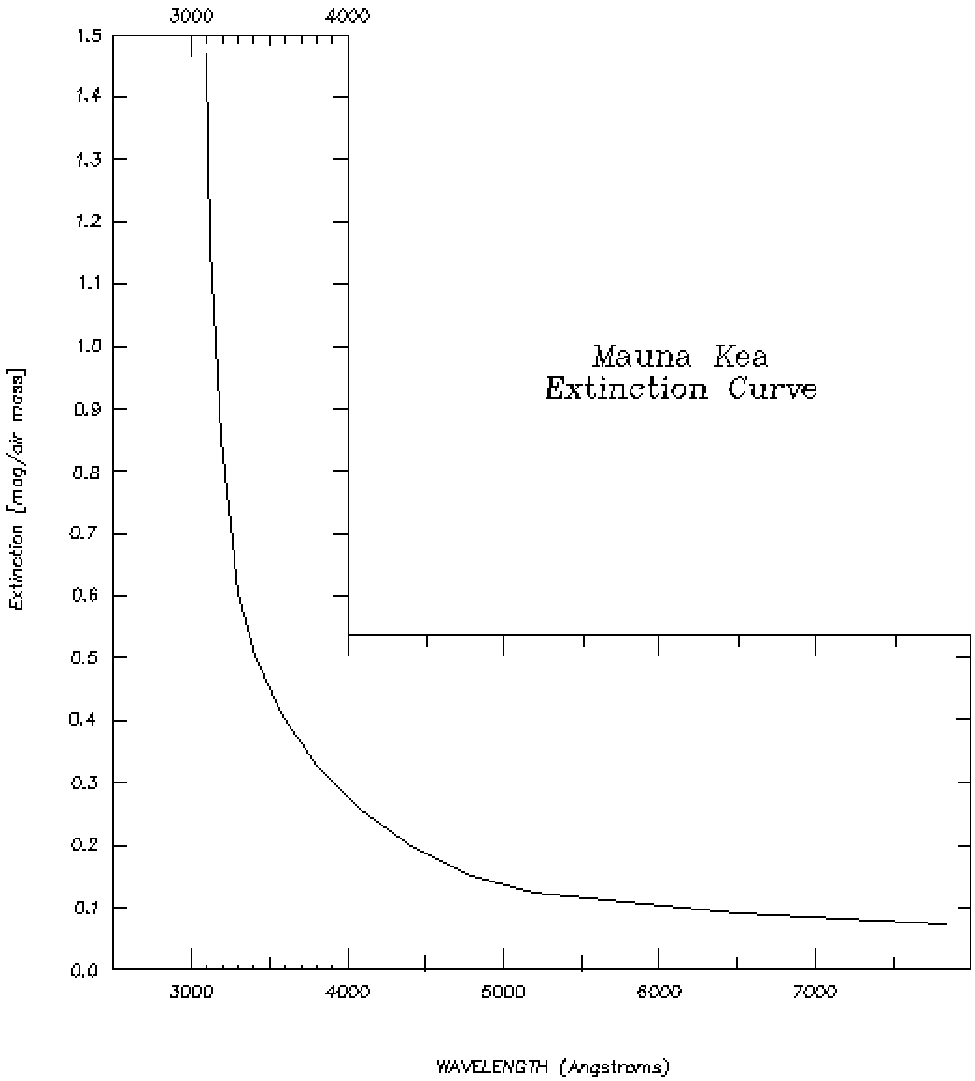
- **Several components contribute to $\tau(\nu)$ in Earth's atmosphere, with different ν dependence and altitude distributions. Extinction effects are illustrated on the next pages.**

TRANSFER IN THE ATMOSPHERE (continued)



Transmission of the Earth's atmosphere in the near-infrared. Absorption here is dominated by strong H_2O bands. Lines show the definitions of the J, H, and K' photometric bands, which lie in relatively clean regions.

TRANSFER IN THE ATMOSPHERE (continued)



Optical-band atmospheric extinction curve (in magnitudes per unit air mass) for Mauna Kea showing the strong increase to short wavelengths. Only the continuous component of extinction is shown.

B. INTERSTELLAR EXTINCTION

- **Interstellar dust is the main source of opacity in the interstellar medium at UVOIR wavelengths.**
- **Dust temperature is so low (usually < 100K) that re-emission in these bands is ~ 0, so $I = I_0\, e^{-\tau_o(\nu)}$.**
- **Dust grains range in size from macromolecules to particles ~ 3000Å diameter.**
- **The grains responsible for UVOIR extinction consist mainly of silicates and graphite (carbon) and produce primarily continuous opacity.**
- **Typical radii and densities for such grains are $r \sim 0.05\mu = 500$ Å and $\rho \sim 3$ gr cm^{-3}.**
- **Optical depth for grains of a given type:**

  $$\tau(\nu) = n_g\, \pi r^2\, Q(\nu)\, L$$

  ...where n_g is the density of grains per unit volume in the ISM, r is the grain radius, Q is the "extinction efficiency," and L is the pathlength

  Q includes both absorption and scattering effects
- **Must sum up over all types and sizes of grains**
- **For bands ~ 0.3–2.5μ, find empirically that extinction is**

  $$\Delta m(\nu) = 1.086\, \tau(\nu) \sim K\, L\, (a + b\, \nu)$$

  ...where K, a, and b are constants
- **Typical extinction in the V-band is about 1 mag per 2×10^{21} gas atoms cm^{-2} in our Galaxy**
- **Any UVOIR photon energy absorbed is re-radiated by the grains at far-IR wavelengths ($> 50\mu$).**