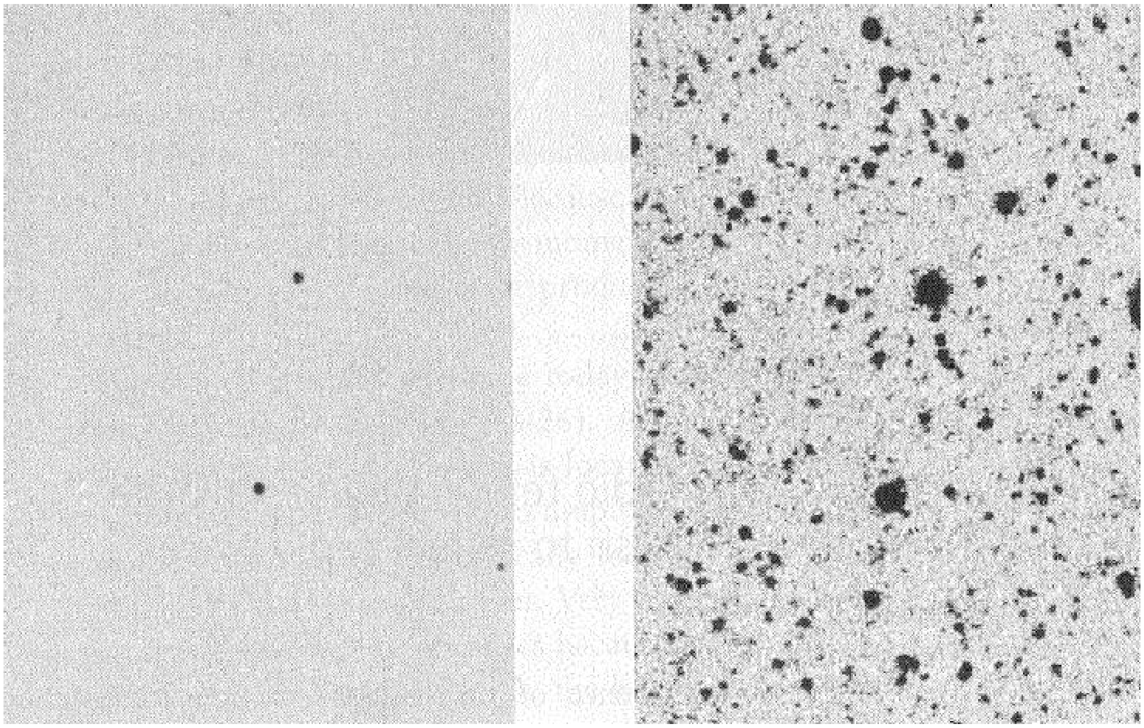
UVOIR FLUX MEASUREMENTS AND SIGNAL-TO-NOISE RATIOS



Comparison of photographic and CCD exposures of same field with 4-m;
Left: 45 min photographic plate; Right: 2 hr CCD (stacked)

Hale's Lament

> **"Starlight is falling on every square mile of the Earth's surface, and the best we can do is gather up and concentrate the rays that strike an area 100 inches in diameter." [G. E. Hale, 1928]**

I. Introduction

In 75 years, we have progressed to 400-inch diameters for optical telescopes, but we still collect a pitifully small fraction of the photons incident from cosmic sources.

This lecture discusses the important considerations in making optimal use of the photons we do have at our disposal.

References:

- **LLM: Chapter 6 on EM signals (though treatment differs from that here)**
- **LLM: Chapter 2 on effects of Earth's atmosphere**

II. The Observer's Problem

The Problem: How to maximize the reliable astrophysical content of observations?

Translates to: How to make <u>accurate</u> flux measurements with a <u>precision or SNR</u> appropriate to the scientific goals given the practical constraints?

Key factors affecting SNR:

- **<u>Source</u>: Luminosity and distance**

  Brightness is usually a strong function of wavelength

- **<u>Destruction</u> or deflection of source photons on way to detection**

- **<u>Noise</u> in process of measuring source photons**

  Telescope <u>size</u>

  <u>Background</u> photons

  <u>Bandwidth</u> (should be maximum consistent with desired information content)

  <u>Instrument</u> throughput, detector sensitivity, noise characteristics

  "Detective quantum efficiency" $= DQE = \left[\frac{SNR_{\rm out}}{SNR_{\rm in}}\right]^2$

  Extent to which equipment can be <u>calibrated</u> to characterize random and systematic errors.

  Real-world <u>time constraints</u> on access to equipment & therefore exposure times

III. Mechanisms for Photon Destruction/Deflection

(Or: Requiem for Photon Demise in Mixed Media)

These effects are mostly not stochastic in character. They must be calibrated for good accuracy in the results and produce systematic errors if this is not done properly.

Almost all are more important at shorter wavelengths

(1) Interstellar extinction (see Lec 15)

Depends on dust grain column density in direction of source

(2) Atmospheric extinction (see Lec 14)

Depends on total atmospheric path length ($\propto sec\,Z$, where Z is the angular distance to the zenith)

(3) Atmospheric refraction

Prismatic effect of differential refraction for $Z > 0$ causes elongation/chromatic separation of point sources

(4) Atmospheric turbulence ("seeing") (see Lec 21)

Causes blurring and jitter of images

(5) Absorption/scattering by optical surfaces

Reflecting and refracting surfaces and transmitting media destroy a large fraction of photons incident on the telescope aperture

IV. Statistical Models of Flux Measurements

Every EM flux measurement is affected by the stochastic processes described in the "Statistics of Observations" pages (Lec 6).

> Every time you try to determine the SNR of a measurement, you are applying an <u>implicit statistical model</u> for the parent distribution.
>
> The appropriate model will differ with the instrument, detector, and waveband. In principle, each system should be analyzed <u>separately</u>. Rules of thumb for one do not necessarily apply to others.
>
> The discussion here is aimed at UVOIR observations made with CCD detectors.

Gaussian Distributions?: We can assume that most stochastic components entering astronomical flux measurements are governed by a Gaussian parent distribution.

> <u>Photon noise</u>, the fundamental source of EM noise, is governed by Poisson/Gaussian statistics in practice.
>
> However, other aspects of photon detection (e.g. amplifier gain variations, seeing variations, dark current, atmospheric transparency changes, etc.) may <u>not</u> be Gaussian. For instance, airglow emission lines usually vary non-randomly. Other elements of a detection system might be governed by Lorentzian or log-normal distributions. For accurate work, it is important to empirically confirm the nature of the statistics governing your measurements.

Multiple Measures: The BEST approach to error estimation is always to make <u>many repeated measures of a value</u>, say $N > 10$. But in practice this may be difficult.

V. Sources of Noise in UVOIR Flux Measures

In the following pages we discuss the major contributors to noise in UVOIR flux measurements. These are

(A) Photon noise

(B) Background noise

(C) Measuring process noise

(D) Other sources of noise

(A) Photon Noise

Fundamental statistical fluctuations in photon arrival times imply that the photon count rate is a random variable (even if the source is strictly constant in luminosity).

- **o <u>Photon arrivals</u> from most astronomical sources are an independent counting process and are described by <u>Poisson</u> statistics (for small mean counts per unit time) or <u>Gaussian</u> statistics (for larger means).**
- **o Photon statistics apply both to source photons and background photons.**
- **o Implications of photon statistics:**

If we consider <u>only</u> noise from photon counting statistics from the source of interest, then

$SNR = N/\sigma(N) = N/\sqrt{N} = \sqrt{N}$, **where N is the total number of source photons counted.**

Known as "square root of N statistics":

N	SNR
100	10
1000	32
10000	100

Because of the contribution of other sources of noise in real measurements, these values are <u>upper limits</u> to the combined SNR.

Caveat on Photon Statistics:

Photons are Bose-Einstein particles, which means that a given quantum state can contain many photons. This gives rise to photon "bunching".

In a photon stream originating in TEQ, the variance in photon arrivals is larger than for a Poisson process:

$$Var(n) = n(1+\delta), \text{ where } \delta = \frac{1}{(e^{h\nu/kT}-1)}$$

The correction term δ is important if $\lambda \gtrsim 2/T$ cm—i.e. in the infrared for normal stellar temperatures.

However, because typical detection systems (optics, detectors) are inefficient, the probability that more than one bunched photon will be detected is small, and the bunching effect can usually be ignored in practice. Photon bunching could be more important in non-thermal sources.

Hanbury-Brown and Twiss (1958) built a special optical "intensity interferometer" which took advantage of the bunching effect to measure the diameters of a small number of hot, bright stars.

(B) Background Noise (Unwanted Signals)

Background light sources are affected by photon statistics but in some cases (e.g. Earth's atmosphere) also by <u>intrinsic variations</u> in flux.

(1) Diffuse Sky Photon Background:

- o Earth's atmosphere: scattered city lights, airglow, aurorae, thermal continuum (IR). Both continuum & emission lines. Emission lines (e.g. [O I] and OH) can be <u>highly variable</u>. Atmosphere is not an issue for space observatories at optical/IR wavelengths. However, skyglow emission lines (Lyα, O I) from residual atmosphere above 500 km are important in far-UV.
- o Moonlight: drastic ($\gtrsim 3$ mag arcsec^{-2}) effect on brightness of optical sky background, depending on phase. Scattered moonlight is blue, so red/IR observations preferred when Moon is bright.

  "Bright time": Full Moon ± 5 days

  "Dark time": New Moon ± 5 days; reserved for faint target astronomy at most observatories.
- o Zodiacal light (sunlight scattered by IP grains); strong direction dependence, but not time dependence; has Solar spectrum. Thermal emission in IR.
- o Galactic background light. In UVOIR, is primarily starlight scattered by IS grains at lower galactic latitudes; has hot-star spectrum but is faint.

  Mid, Far-IR ($\lambda > 20\mu$) emission from warm dust: "IR cirrus"

SKY BACKGROUND

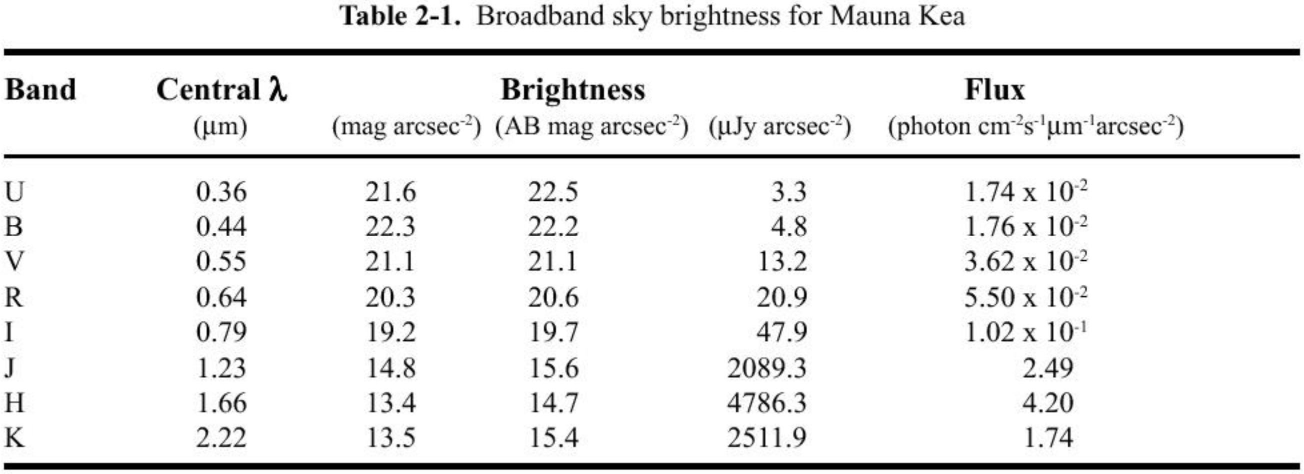


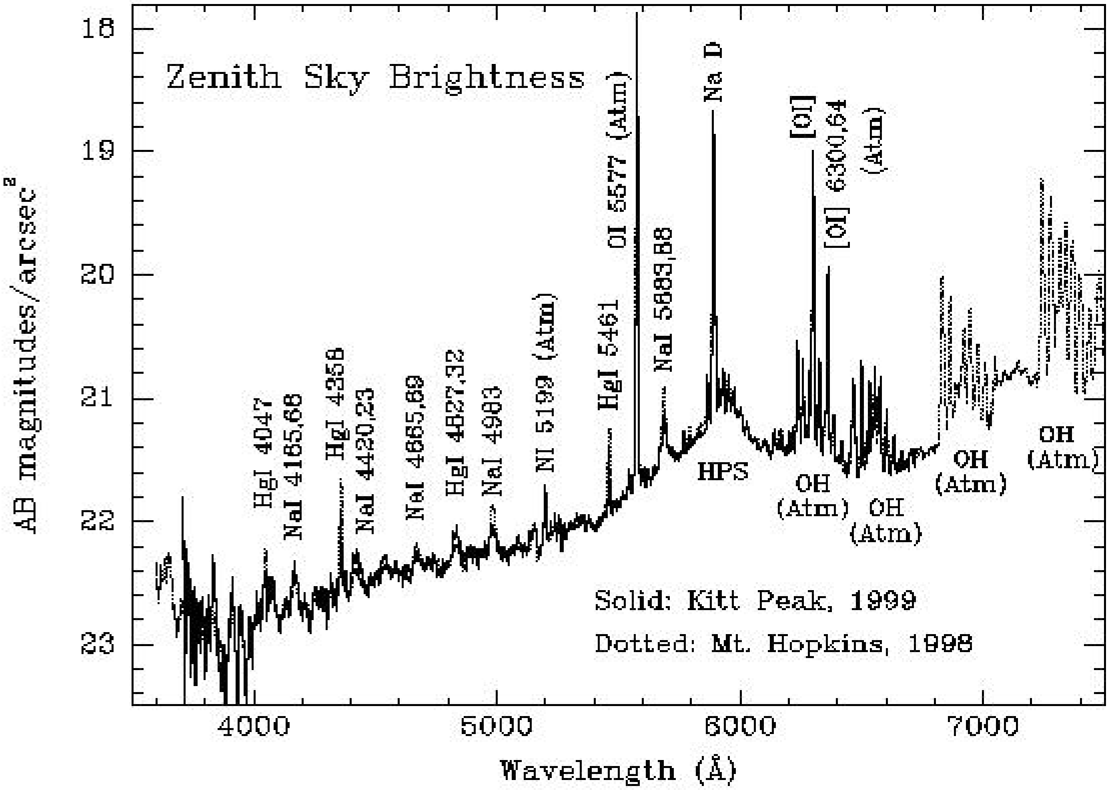
Table 2-1. Broadband sky brightness for Mauna Kea

Band	**Central λ**	**Brightness**			**Flux**
	(µm)	(mag arcsec^{-2})	(AB mag arcsec^{-2})	(µJy arcsec^{-2})	(photon cm^{-2}s^{-1}µm^{-1}arcsec^{-2})
U	0.36	21.6	22.5	3.3	1.74×10^{-2}
B	0.44	22.3	22.2	4.8	1.76×10^{-2}
V	0.55	21.1	21.1	13.2	3.62×10^{-2}
R	0.64	20.3	20.6	20.9	5.50×10^{-2}
I	0.79	19.2	19.7	47.9	1.02×10^{-1}
J	1.23	14.8	15.6	2089.3	2.49
H	1.66	13.4	14.7	4786.3	4.20
K	2.22	13.5	15.4	2511.9	1.74

Broad-band (continuum & line emission) sky background levels (no Moon) at Mauna Kea

Note dramatically increased brightness for JHK bands.

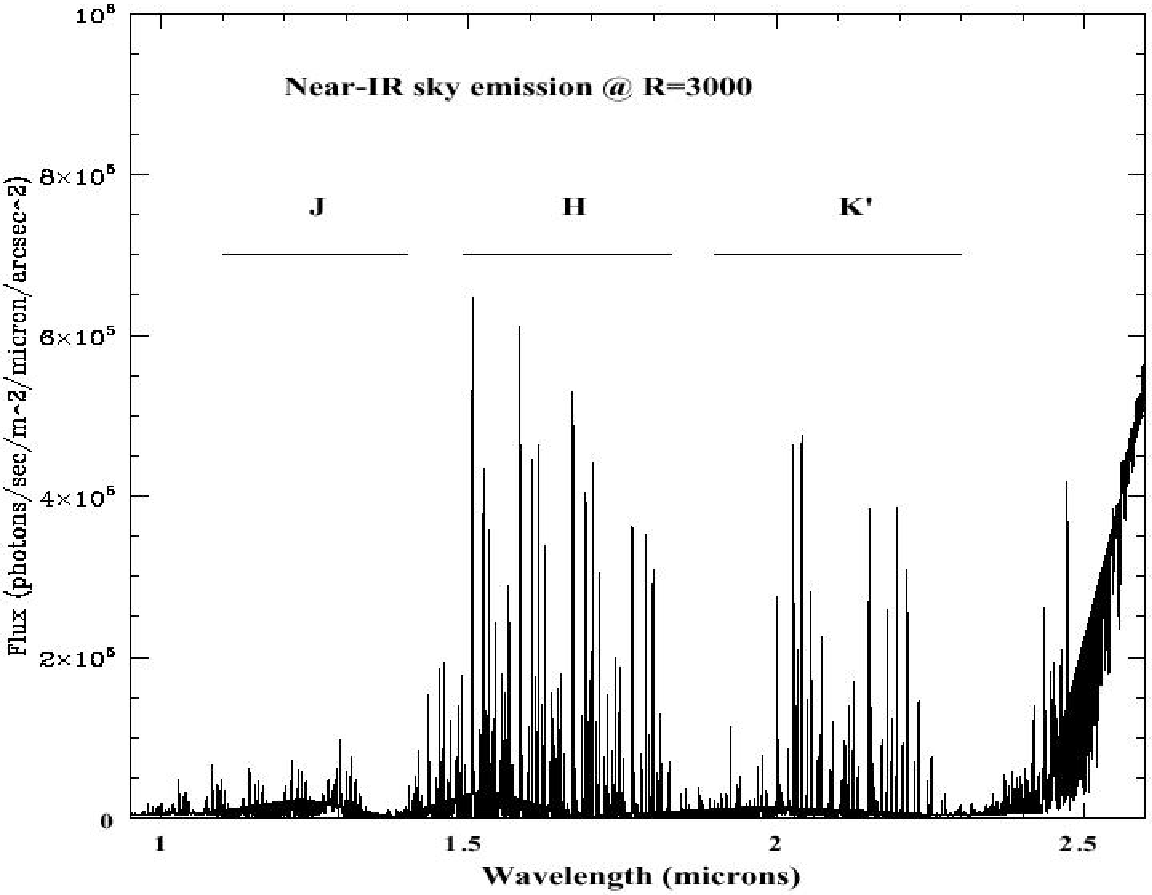
SKY BACKGROUND



Night sky spectrum from KPNO

Shows red continuum, Hg, and Na emission lines from scattered city lights. (HPS = "high pressure sodium" lamps). Strong [O I] lines are auroral. Region redward of 6200 Å shows start of forest of upper-atmospheric OH lines, which continues through near-IR.

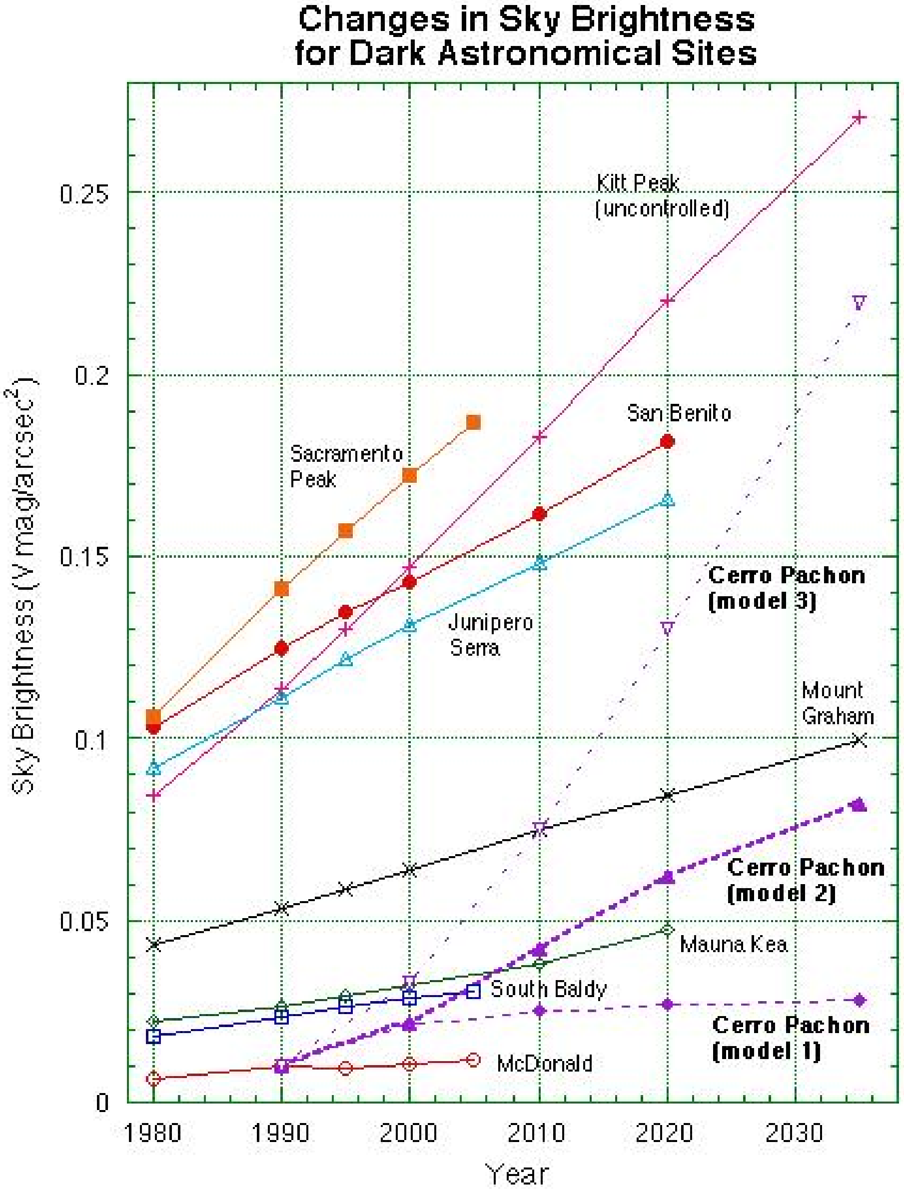
SKY BACKGROUND



Night sky emission lines (mainly OH), near-IR

Shows continuation of atmospheric OH spectrum from preceding KPNO plot. OH forms at 75 km altitude, so affects all ground-based sites. Impact of lines is devastating for certain kinds of observations. Natural extra-atmospheric background at these wavelengths is up to 1000 times fainter.

SKY BACKGROUND



Growth of city light contamination (A. Walker)

No sites are free of serious & increasing light pollution except McDonald Observatory (in west Texas, where apparently nobody wants to live). Of special interest for us at UVa are Mount Graham and Sacramento Peak.

SKY BACKGROUND

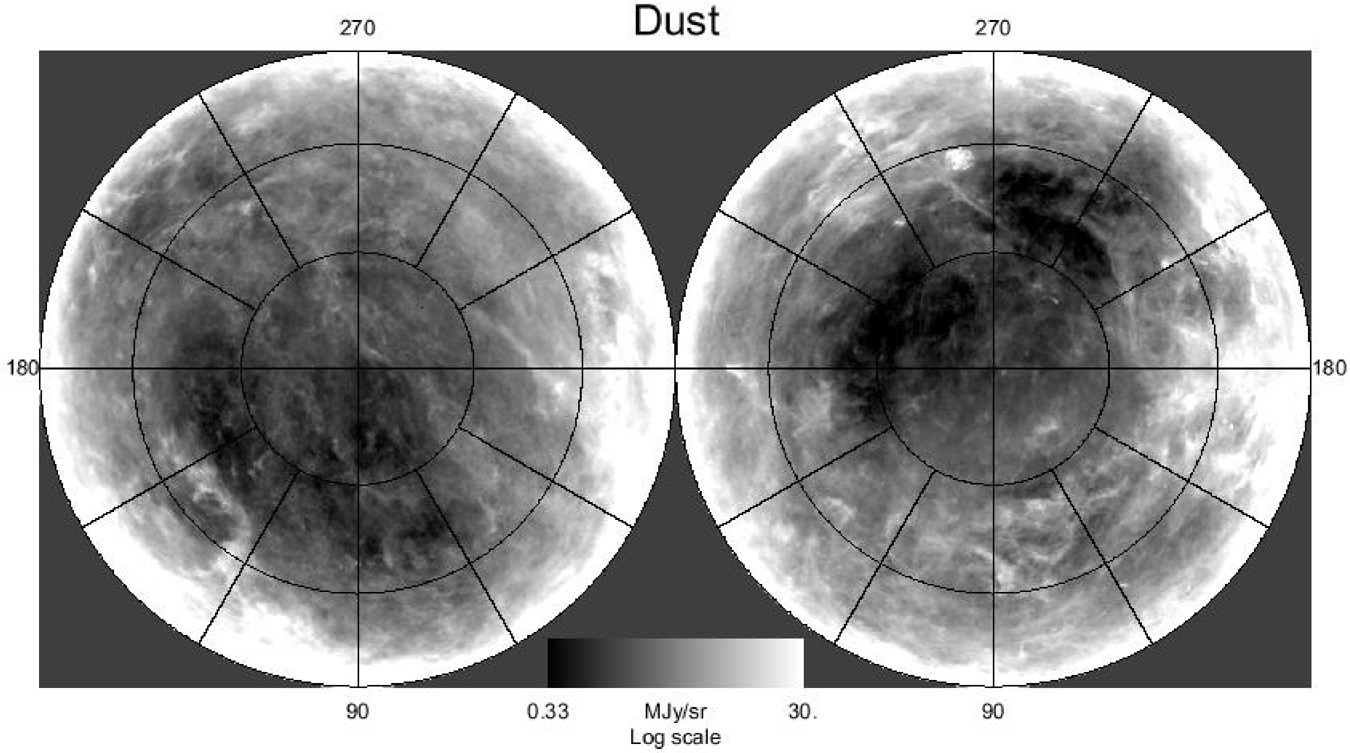


FIG. 8.—Full-sky dust map for the NGP (*top*) and SGP (*bottom*)

Hemispheric images of far-infrared "cirrus" (COBE/DIRBE)

"Cirrus" emission is produced by warm interstellar dust grains at typical distances of 100-3000 pc within the Galaxy. Far-IR (50–200 μ) observations can be importantly affected by this strongly non-uniform background. Must make good determination of local 2D cirrus structure in order to remove its effects.

BACKGROUND NOISE (cont)

(2) Discrete Cosmic Sources

- o Field stars (e.g. scattered light from bright stars; produce an "exclusion zone" around stars)

  Star light scattered, refracted, or diffracted by the atmosphere and by telescope optics and structures can produce effects at large distances from a star. King (1971) showed that the profile of a star image has a Gaussian core, but then an exponential shoulder and a power law at $r > 30$ arcsec.

- o Extended envelopes from nearby galaxies
- o Faint distant galaxies (serious problem at faint levels since are thousands per square degree)
- o "Confusion" caused by source blending within spatial resolution cell. More serious in radio astronomy, but a major UVOIR problem in some cases, e.g. star clusters

(3) Detector Noise

- o Chemical fog (photographic emulsions)
- o "Dark current": thermal emission in absence of signal; major problem; requires detector cooling

  For a semiconductor such as a CCD array, the dark current behaves as: $\dot{n}_{dark} = AT^{3/2}e^{-E_g/2kT}$, where E_g is the band gap energy and T is the temperature. (See Lec 11 for discussion of semiconductors.)

  Is a more serious problem for IR detectors because of smaller band gaps.

- o Cerenkov photons
- o CCD's: variations in electronic "bias"

BACKGROUND NOISE (cont)

(4) Telescope Backgrounds

- IR: emission ($T \sim 280$ K) of optics and other structures visible to detector ($\lambda \gtrsim 1.5\mu$)
- Diffraction & scattering (e.g. from dust on optics) contributes to background at all λ's

Note the ADVANTAGE of 2D digital array detectors for background determination:

Ordinarily 2D devices provide a large number of background samples surrounding a source of interest.

The samples are also usually obtained (in imaging, for instance) simultaneously with the source observations, which is very important in the case of large & variable backgrounds. As long as the background contains only low spatial frequencies, it can be well modeled and removed, greatly improving the detection of faint sources.

The 2-D advantage is especially important in the near-IR, where the sky, telescope, and detector backgrounds can be fierce.

NOISE IN UVOIR MEASUREMENTS (cont)

(C) Measuring Process Noise

(1) Amplifier noise

- **E.g.: CCD "readout noise" (RON) produced by gain variations in on-chip amplifiers**
  - Usually quoted as "equivalent electrons" ($n_{\rm RON}$) of rms noise per pixel
  - Implies additional variance of $n_{\rm RON}^2$ per pixel
  - Adds as much variance to signal as $n_d = n_{\rm RON}^2$ detected photons
  - Is <u>independent</u> of integration time whereas ratio of photon and dark current noise to signal is reduced for longer integration times
  - If RON is important, want to minimize # of readouts

(2) Gain variations in electron multipliers (e.g. PMTs, image tubes, microchannel plates) and electronic readout devices (e.g. delay line anode grids)

(3) Microdensitometer readout gain variations (Pg plate digitization)

(D) Other Sources of Noise

(1) Sensitivity variations across 2-D detectors

- **Grain noise in Pg plates (grain size $\sim 25\mu$)**
- **"Flat field" effects (pixel-to-pixel sensitivity variations) in CCDs and other semiconductor devices. For CCDs, typically of order ~5% with a strong wavelength dependence.**

- o **Extreme "hot" or "cold" pixels in array detectors; cosmetic defects, e.g. bad columns. Worse in IR detectors. Can be induced by cosmic ray damage.**
- o **Intra-pixel variations; can be important if point spread function is undersampled by pixels (as in high-resolution space telescopes), but is technically difficult to measure.**

(2) CCD's: charge transfer inefficiencies (worsened by cosmic ray damage)

(3) Variations in atmospheric turbulence/seeing

(4) Strong and/or variable absorption in atmosphere (general extinction; clouds; bands of H_2O, O_2, O_3)

(5) Cosmic rays: tracks easily detected in CCD, other devices; produce serious, though localized, effects; incidence is governed by Poisson stats.

(6) Interference (light leaks, TV, radio, radar, cell phones, etc)

(7) Radioactive decay glow in filters/windows

(8) Guiding errors

(9) Mechanical flexure in telescope or instrument; focus shifts

NOISE IN UVOIR MEASUREMENTS (cont)

NOTE on the Near-Infrared "Quadruple Whammy":

If you were paying attention to the earlier material, you will realize that this spectral region (0.8–3 μ) is affected by multiple difficulties, including:

> (i) Bright sky and telescope background continuum emission; (ii) bright atmospheric emission lines; (iii) strong atmospheric absorption lines (mainly H_2O); and (iv) high detector dark currents.

In the past, the problem was compounded by low sensitivity detectors and the absence of large format 2D detectors. Older, single-element IR detectors required that telescopes be capable of rapid "chopping" (spatial offsets) between target and nearby sky.

The advent of high QE, 2D detectors for the NIR has meant a terrific improvement in performance, among other things usually eliminating a requirement for telescope chopping.

However, the presence of bright & variable OH emission lines, as well as widespread H_2O absorption, still greatly complicates spectroscopy of faint targets in the NIR.

VI. Example: Effects of Background on SNR In Fixed Aperture Photometry

- **Application: photometry with a photomultiplier tube (one detection element) or from a 2-D image array using a virtual aperture of fixed size. Consider only photon statistics.**
- **Measure a compact source ($<$ aperture size) and background nearby with the same-sized aperture**
- **Yields two measurements: "on source" and "background" with total detected photon counts T & B, respectively. What is SNR?**
  - **o Net signal: source count $S = T - B$**
  - **o Variance?**

    $$Var(S) = Var(T) + Var(B) = T + B = S + 2B$$

  - **o Then $SNR \equiv S/s_{\bar{S}} = \frac{S}{\sqrt{S+2B}}$**

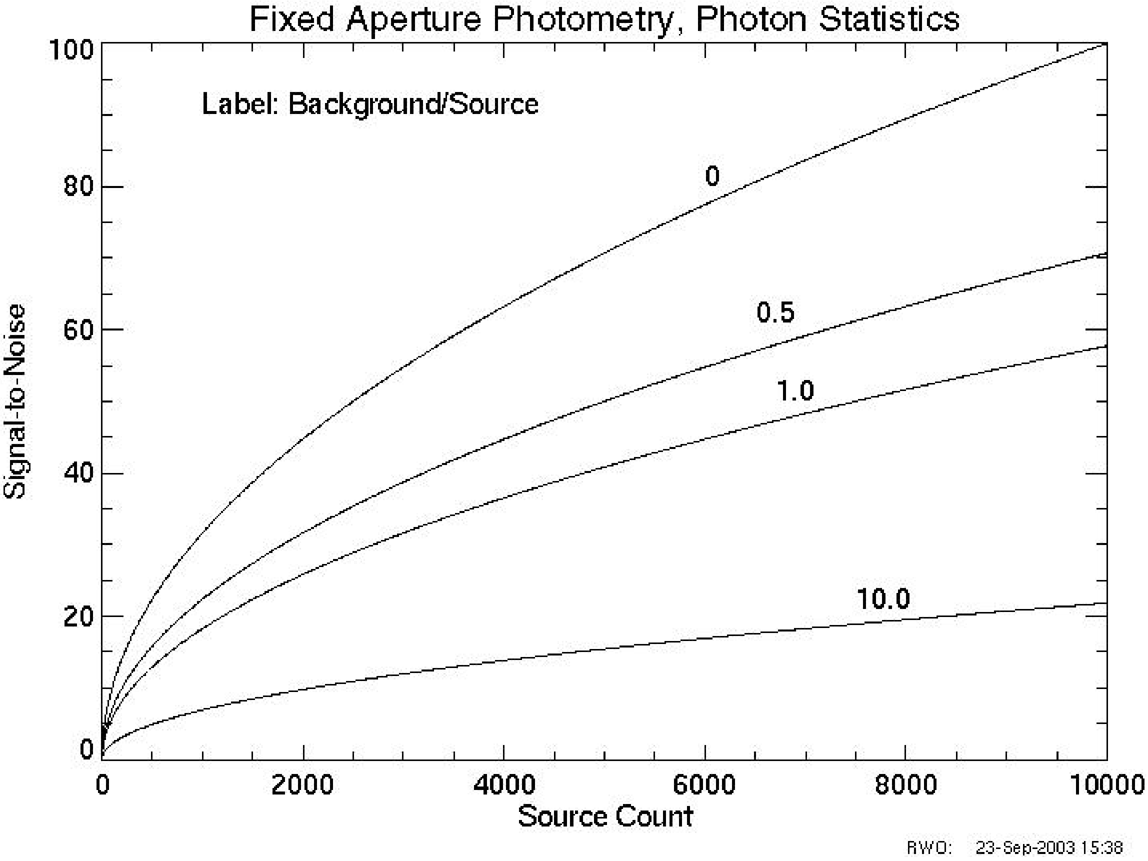
    NB: the background term enters twice because both measurements include the background

  - **o Re-write as a function of S, B/S:**

    $$SNR = \frac{\sqrt{S}}{\sqrt{1 + 2\,\frac{B}{S}}}$$

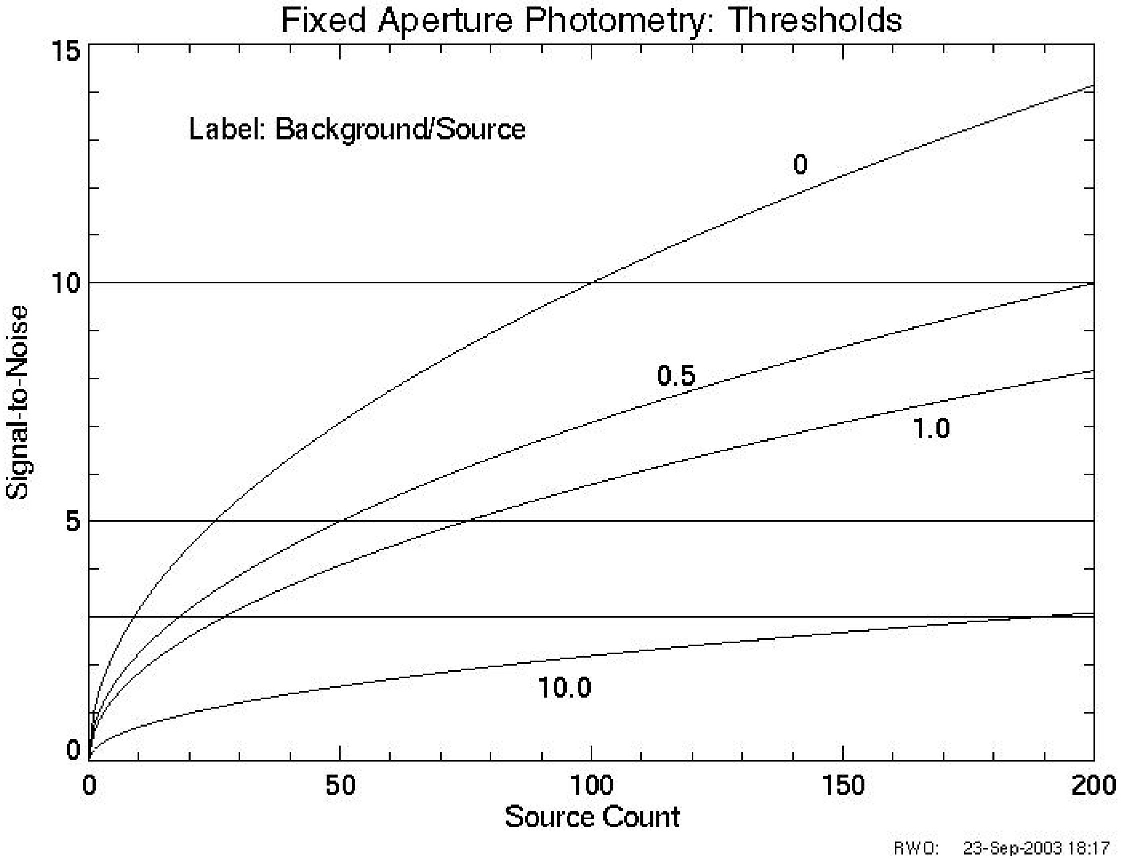
  - **o Plot result (see next page)**
  - **o Limiting behavior:**

    "Source limited": $S >> B \Longrightarrow\ SNR = \sqrt{S}$
    "Background limited": $B >> S \Longrightarrow\ SNR = \frac{S}{\sqrt{2B}}$



Signal to noise ratio as function of source count and background ratio in fixed aperture photometry

In near-IR applications, can easily find situations where $B/S > 100$.



"Threshold detections" in Fixed Aperture Case

Follow line for chosen $SNR_{\rm min}$ to obtain the minimum number of source counts required for different background ratios. X-ray astronomers would consider SNR = 3 to be a threshold detection, but UVOIR astronomers tend to be more discriminating (SNR of 5 to 10).

VII. Example: SNR in Photometry With Large Background Sample

- **Assume two different aperture sizes: one (N pixels) containing all the light of a compact source and one (M pixels) containing only the background. M can be very large.**

  E.g. DAOPHOT uses a small circular aperture for the source and a large circular annular aperture for the background, though the shape is not relevant to this derivation.

- **Let $\bar{b}$ be the mean background count per pixel. If B is the total background count in the large background aperture, then we can estimate $\bar{b}$ and its variance as follows:**

$$\bar{b} = B/M \text{ and } Var(\bar{b}) = \frac{1}{M^2}(B = M\bar{b}) = \frac{\bar{b}}{M}$$

- **We estimate the source count as $S = T - N\bar{b}$. Then:**

$$Var(S) = Var(T) + Var(N\bar{b})$$

$$Var(S) = T + N^2 Var(\bar{b}) = T + \frac{N}{M}N\bar{b}$$

$$Var(S) = S + N\bar{b}\left(1 + \frac{N}{M}\right)$$

$$\Longrightarrow \quad SNR = \frac{S}{\sqrt{S + N\bar{b}\left(1 + \frac{N}{M}\right)}}$$

- **Get same result as (VI) if $M = N$ but can reduce coefficient of background term from 2 to 1 if $M >> N$.**

VIII. Effect of Exposure Time and Readout Noise on SNR

In preceding examples, the quantities entering the SNR calculation are proportional to the exposure time. I.e. in the fixed aperture case:

$$S = \dot{S}t \quad \text{and} \quad B = \dot{B}t$$

where t is the total exposure time of the observation, and $\dot{S}$ and $\dot{B}$ are the rates at which source photons and background photons, respectively, are detected. This implies:

$$SNR = \frac{S}{\sqrt{S + 2B}} = \frac{\dot{S}\sqrt{t}}{\sqrt{\dot{S} + 2\dot{B}}}$$

Interpretation: Although the noise and the signal both increase as t increases, the signal increases faster, hence SNR improves...but not in direct proportion to t.

Improvement of SNR in proportion to $\sqrt{t}$ is a consequence of photon statistics and applies generally to any situation where they dominate measurement uncertainty. (Also applies to sources of detector noise—e.g. dark current—which are characterized by Poisson statistics.)

> From a practical perspective, this can be viewed as a slow improvement of SNR for a given investment of time. When observing time is at a premium, there will be a fairly obvious point of diminishing returns.

However, this does not apply to a large class of detector noise which is <u>independent</u> of integration time. Detectors with this behavior are sometimes called "Class II" detectors. Photographic emulsions, bolometers, infrared array detectors, and CCDs are all in this category.

In CCDs <u>readout noise</u> is independent of integration time and in some cases is an important constraint on the SNR.

To illustrate the effect of readout noise, rewrite the result for the fixed aperture case as follows:

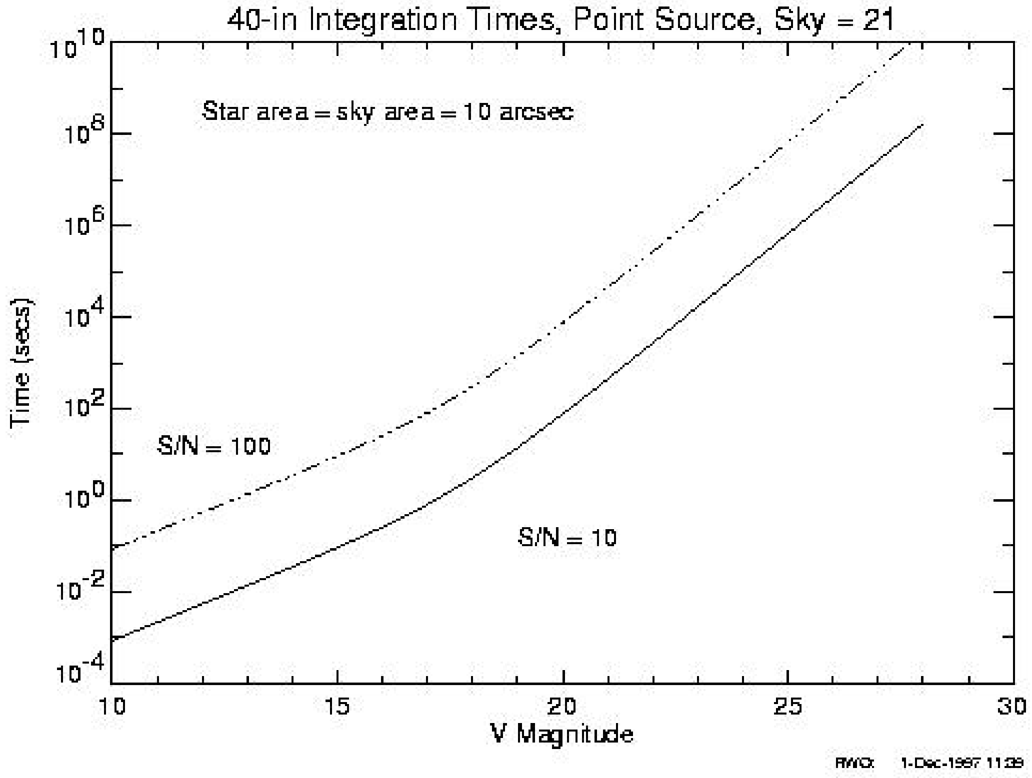
$$Var(S) = \dot{S}\,t + 2N(\dot{b}\,t + \dot{d}\,t + n_{\rm RON}^2)$$

where N is the number of pixels in the measuring aperture, $\dot{b}$ and $\dot{d}$ are, respectively, the detected background photon rate and the dark count rate per pixel, and $n_{\rm RON}$ is the readout noise per pixel (quoted in numbers of equivalent electrons of noise). A single readout after exposure time t is assumed.

The resulting signal-to-noise is then:

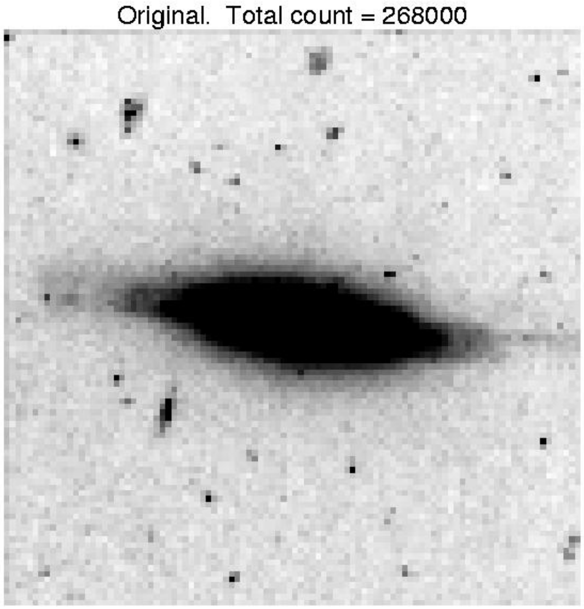
$$SNR = \frac{\dot{S}\sqrt{t}}{\sqrt{\dot{S} + 2N\left(\dot{b} + \dot{d} + \frac{n_{\rm RON}^2}{t}\right)}}$$

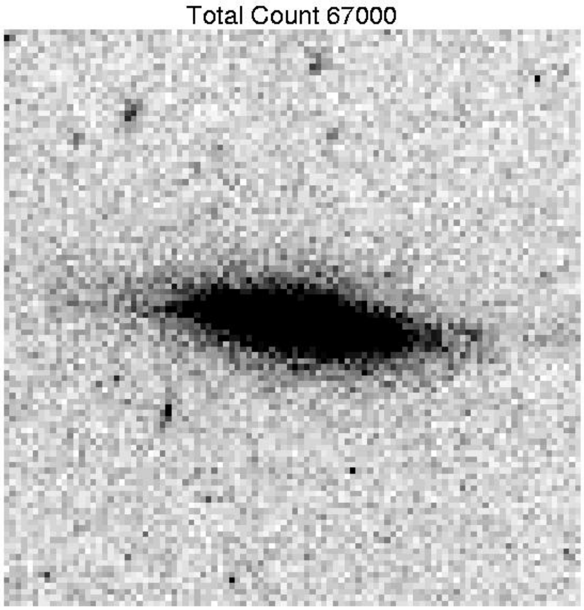
When RON is important, one wants to <u>minimize the number of readouts</u> for a given total exposure time and also to minimize the <u>number of pixels</u>, N, covering the region of interest.



Example estimate of integration times for the Fan 40-in CCD imaging system including the effects of source noise, sky background, and readout noise.

Note change of slope at $V \sim 18$, which is the transition between background noise dominance (for fainter sources) and source photon noise dominance (brighter). Diagrams like this allow you to optimize your observing program by making trades-off between SNR, source brightness, and integration time.





Effect of SNR on appearance of a digital image

Upper image has SNR 2.3X that of lower image.
The lower image is a Poissonian sample using the upper image as a parent distribution.