

PROBLEM SET 1
DUE WEDNESDAY SEPTEMBER 10

Please use *cgs* units and the standard symbols defined in class in working problems. Define any new symbols used, and work problems algebraically *before* plugging in numbers. Show all your work, and remember to give the units of all answers. Please write clearly and use only one side of each page.

Especially in the early sets, the problems are mostly easy. Two tips for avoiding simple mistakes: (1) read and follow the directions carefully; (2) be careful in converting to *cgs* units if you are used to working in *MKS*.

Problems will be graded on a semi-pass/fail basis. You will receive 2 points for getting a problem substantially correct (even with some numerical errors or conceptual lapses) and 1 point for making a serious attempt. (Simply writing down basic equations doesn't count as "serious," unless these are non-obvious.) The two-point maximum applies to each labeled part of a problem.

Most of the problems in PS1 involve the "magnitude" system, as defined in Lecture 2.

- (1) Some conclusions about cosmic objects can be reached without making observations. For instance, we know that the spectral energy distribution of the Sun cannot be a power law like $L_\nu = k\nu^{-1}$ over all electromagnetic frequencies, where L_ν is the luminosity of the Sun per Hz and k is a constant. How do we know this?
- (2) Estimate the apparent V magnitude of a 10-km diameter asteroid lying in the same orbit as Neptune. (Asteroids shine by reflected sunlight and have a typical albedo of 10%.) How close would such an object have to be to Earth to be visible to the naked eye?
- (3) A typical globular star cluster might contain 2×10^5 stars like the Sun within a radius of about 3 pc and lie at a distance of 5000 pc from the Sun. Estimate the mean V-band surface brightness of such a cluster (in magnitudes per square arcsec). Would such an object present a large contrast against a typical ground-based sky background of $\mu_V = 21$ mags per square arcsec ("large" here would be a minimum of 10:1 in flux ratio)? Does your answer depend on the distance to the cluster?
- (4) Show that the definition of the magnitude system offers at least one convenient feature: namely that the change of magnitude caused by a given (small) fractional change in the flux of a source is numerically equal to the fractional change. (This implies, for instance, that a 2% error in the measured flux of a source converts to a ~ 0.02 mag difference.)
- (5a) Suppose you are counting sources as a function of apparent brightness from a population which is uniformly distributed throughout space with a density n_1 . All the sources have the same luminosity, L_1 , in the V band. Derive an expression which gives the number of sources brighter than a given V magnitude m . Show that this scales in proportion to $10^{0.6m}$.
- (5b) If we lived in a homogeneous spherical galaxy with the same mean density of stars as

that of the Milky Way's disk near the Sun, approximately how many stars would there be brighter than $V = 15$?

ROMPs

Rough Order of Magnitude Problems (ROMPs) are those where you need to fill in essential concepts and quantities from memory or an informed guess but where the answers need not be good to better than a factor of 10. The classic ROMP was posed to physics students by Enrico Fermi: "How many piano tuners are there in Boston?"

Facility with ROMPs is an important skill to develop. They are good practice for real life as a scientist (and for the Qual Exams). ROMPs in 511 problem sets will often be only tenuously related, or even unrelated, to the material already covered in lectures. This is deliberate.

In answering ROMPs throughout the course, you should work *entirely on your own*. You should not consult the text, your notes, other references (printed or electronic) or other people. If you have questions, ask me rather than your colleagues. Work the problems based on "common sense" and facts that you already know. Make reasonable assumptions about the various quantities which enter the problem, and *explicitly state what these are in your writeup*. Clearly describe your reasoning. You should always give a mathematical expression for the solution before plugging in numbers.

- (6a) How long would you have to integrate on a $V=0$ magnitude star with the Keck 10-m telescope in order collect enough energy to boil a cup of coffee? Recall that the flux density of a zeroth magnitude star is $f_\nu = 3.6 \times 10^{-20} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$ or 3630 Janskys. Assume that the energy collection is 100% efficient, that you use a bandwidth of 10% of the frequency, and that there is no heat loss from the coffee during the integration.
- (6b) How would the answer change for the case of a 3630 Jansky radio source observed at 1 GHz by the 100-m diameter Green Bank Telescope? [NB: the brightest radio source at 1 GHz other than the Sun is Cas A, with $f_\nu \sim 3300 \text{ Jy}$.]
- (7) How much data storage (in bytes) would be required for an all-sky, single-band (say at 6 cm wavelength), flux survey with the 100-m diameter GBT? Assume the detector is a single-element radiometer.
- (8) Use the fact that the detection threshold for stars with the unaided human eye is about 6th magnitude to estimate the flux received at the Earth from a zeroth-magnitude star. Give your answer in units of $\text{erg s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1}$. *Hint*: light consists of photons.

NB: the answer to this problem (in different units) is actually quoted in problem (6) above. Here, you are expected to devise a sensible ROM method of estimating that answer. You should be able to get within about one order of magnitude of the known answer.