ASTR 511 (O'Connell) **Fall 2003**

PROBLEM SET 2

DUE WEDNESDAY SEPTEMBER 24

Please use *cgs* units and the standard symbols defined in class in working problems. Define any new symbols used, and work problems algebraically *before* plugging in numbers. Show all your work and remember to give the units of all answers. Please write clearly and use only one side of each page. 2 points maximum for each labeled part of a problem.

Most of these problems involve the use of the Planck function (Lecs 2 and 4) to approximate the emergent energy distribution of stars.

(1) The specific intensity of radiation crossing the surface into the outbound hemisphere of a source in thermal EQ is $I(\nu, \theta) = B(\nu)$, independent of θ, where $B(\nu)$ is the Planck function. Show that the flux of radiation crossing the surface is $F(\nu) = \pi B(\nu)$.

Now derive an expression for the luminosity at any frequency of a spherical source in thermal EQ in terms of $B(\nu)$ using the result you just obtained. Then, using the definitions of observed flux and specific intensity, show that the specific intensity observed at Earth from the source is $I_{\oplus}(\nu) = B(\nu)$. (Assume that the source is fully resolved by the optical system employed and no absorption occurs along the optical path.)

(2a) Show that if a source with an incident flux at Earth of 1 Jy is observed at any electromagnetic frequency ν with a bandwidth of k% of the frequency, the resulting photon rate ϕ per square centimeter per second at the telescope aperture is a constant. Derive the constant.

(2b) Derive an expression for converting source monochromatic magnitudes per Å at any wavelength to fluxes measured in Jy's. The limiting flux of the JWST infrared mission (now under development) is estimated to be 1 nanoJy at 1μ wavelength. To what m_λ does this correspond?

(2c) To what distance (in parsecs) could an individual massive main sequence star like Spica (M_V = 0.98, T_e = 23000K) be detected with JWST at 1μ? Assume Spica radiates like a black body and use the applicable approximation for B_λ for the temperature and wavelengths involved.

(3a) In doing problem (2) of Problem Set 1, you probably derived the mean "monochromatic luminosity" of the Sun in the V-band. You should have obtained $L_\lambda^\odot = 5.10 \times 10^{29}$ erg s^{-1} Å^{-1}. Re-derive this now if you did not obtain this result. Then, use the radius of the Sun to determine the mean monochromatic flux of the Sun at its surface in the V-band (in units of erg s^{-1} cm^{-2}Å^{-1}).

Compare this answer to a Planck law (using your result in problem 1) to estimate the corresponding equivalent black body temperature T_V of the Sun in the V-band. Compare your result to the known effective temperature, T_e, of the Sun. [Tip: it's easy to do the computations in IDL if you use the AstUseLib routine "planck.pro".]

(3b) Use your results in (3a) to compute the integrated luminosity of the Sun (in units of ergs s^{-1}) within the V band. You can assume the V filter has a top-hat response function with a width of 880 A. Estimate the bolometric luminosity of the Sun using the T_V you obtained in (3a). What fraction of the bolometric luminosity of the Sun emerges in the V band?

(4a) So-called "brown dwarfs" might have R = 0.1 $R_\odot$ and T = 1500 K. If you had at your disposal a telescope with a monochromatic limiting magnitude at the K band (2.2μ) of m_λ^{lim} = 20, estimate the maximum distance (in parsecs) at which you could detect a brown dwarf. Assume the BDs radiate like black bodies.

(4b) If the spatial density of brown dwarfs is 1 pc^{-3}, how many are within range of your telescope, given the assumptions in (4a)? If the limiting magnitude were = 22 instead, how many would be within range?

(5a) Fill out the following diagram for spatially-resolved, thermal sources. Plot the apparent magnitude of a source in the V-band as a function of its temperature (range 10^3–10^5K) and its angular diameter. Make the abscissa of the plot $\log T$ and the ordinate m_λ. Plot lines for the following values of the angular diameter: 0.003″, 0.01″, 0.03″, 0.1″, 0.3″, 1.0″. (It is easy to use IDL to make the plot.) Give analytic expressions before you run numerical calculations.

(5b) Look up the apparent magnitude and surface temperature of the star Betelgeuse (α Ori). Then use your diagram from (5a) to determine its angular diameter. Is Betelgeuse resolvable (or nearly) with current technology? If Betelgeuse is at a distance of 200 pc, what is its linear radius in units of the Sun's radius?

(6) The "broad line" region of an active nucleus contains rapidly moving emission line gas which is assumed to be in gravitational orbits near the supermassive black hole. If the Doppler width of the broad lines is denoted v_{1000} in units of 1000 km s^{-1}, and the mass of the central black hole is M_9 in units of $10^9 M_\odot$, estimate the angular diameter of the broad line region for a galaxy at a distance of 10 Mpc. Is this region resolvable (or nearly) with current technology?

ROMP. Review the expectations for working on ROMPs as discussed in PS-1. Remember to explicitly state assumptions.

(7) It is claimed that there are more stars within the observable universe than there are grains of sand on all the beaches on the Earth. Is this correct?