YISHENG

TU

- An analytic solution to the three-body-problem
- Actual topic;: How to give a bad talk

The nature of a three body is the interaction between three bodies, such as the interaction between the Earth (the object that we are sitting/standing on), moon (the bright circular thing you see at night sometimes, each month (approximately 30 days) the appearance of the moon repeats) and the sun (the object you observe in the sky during a period known as “day”, which is usually the period starting somewhat before you waking up and ends before you going to bed. Speaking of the sun, the apparent magnitude of the sun is about -26, and it varies due to stellar activities such as stellar wind, magnetic breaking). The interaction we are interested in is gravitational interaction. Gravity is the force that pulls everything down. By “down” I mean the direction pointing from your head to feet but if you are upside-down it can also from your toe to your head. Everything in the universe is attracted to every other thing due to gravity, although the force can be ignored at very small scales, such as the scale of the classroom that we don’t feel the gravity of each other. At smaller scales, for example at atom scales, the dominant force are electric-magnetic force and weak force. It usually believed that the carrier of these forces are “photons”, which is what astronomers, defined to be the people that do astronomy (a subject that studies objects in the sky and it’s a subject probably you guys are familiar with), collect using telescopes.

Telescopes are inventories that are used to observe objects far away. There are two kinds of telescopes: reflective and refractive. The ones at APO (that most of us visited) are reflective. By “reflective” I mean it reflects light emitted from distant object and converge them onto some device, usually CCD or CMOS, although in general CMOS is much cheaper than CCD.

Here is a question for all of you: If I’m moving at 150 mph on a highway, and a truck is 100 miles ahead of me moving at 50 mph, assume it’s straight rode and assume I can keep my speed a constant and so does the truck. Ignore friction between the cars and ground, as well as the consumption of gas. What will be the time, if all the preceding conditions are met, does a cop catches me speeding? A related question is what is in the truck that makes it move at 50 mph? Please do not discuss with yourself or your neighbor, as any communications, including one with oneself, will be interpreted as cheating and no cheating is tolerated during this talk.

WARNING: PLEASE LOOK AWAY
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Now back to the three-body problem. The gravitational attraction between two bodies is given by

$$F = -\frac{Gm_1m_2}{r^2}$$

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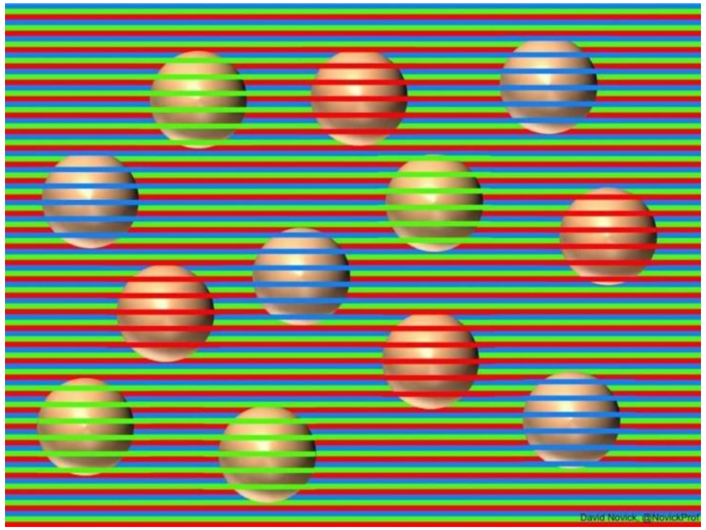
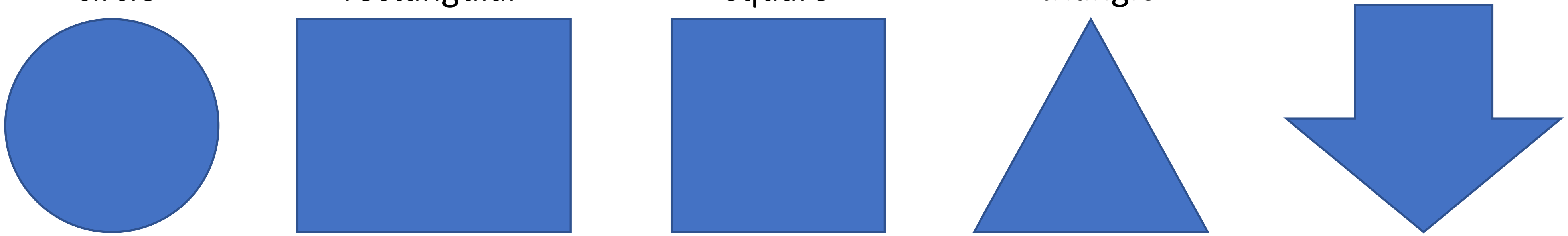


Let’s learn shapes!

circle rectangular square triangle arrow



Pointing towards the direction of gravity

Question: What’s the title of this talk?

Pay attention to this!

Gravitational attractionOMIT

$$F = -\frac{Gm_1m_2}{r^2}$$

Three ♭ody pro♭lemOMIT

Interaction ♭etween three ♭odiesmi

Letns put them into some e×ercise

The Keplerian Hamiltonian of 2BP around resonance at $\Lambda_i = \Lambda_{i,0}$ in Poincare coordinate

$$\mathcal{H}_{kep} \approx -\frac{1}{2}\left\{\frac{1}{\Lambda_{1,0}^2} - \frac{2I_1}{\Lambda_{1,0}^3} + \frac{3I_1^2}{\Lambda_{1,0}^4} + \frac{\nu^3}{\Lambda_{2,0}^2} - \frac{2\nu^3 I_2}{\Lambda_{2,0}^3} + \frac{3\nu^3 I_2^2}{\Lambda_{2,0}^4}\right\}$$
$$= -\frac{1}{2} + I_1 - \frac{3}{2}I_1^2 - \frac{\nu^3}{\nu^2\alpha_0^{-1}} + \frac{2\nu^3 I_2}{\nu^3\alpha_0^{-3/2}} - \frac{3\nu^3 I_2^2}{\nu^4\alpha_0^{-2}}$$
$$= -\frac{1}{2} - \frac{\nu\alpha_0}{2} + I_1 - \frac{3}{2}I_1^2 + \alpha_0^{3/2} I_2 - \frac{3}{2}\nu^{-1}\alpha_0^2 I_2^2$$

Add perturbation, define some simple terms

$$\begin{array}{ll} \theta_1 = j\lambda' + (k-j)\lambda - k\gamma & \Lambda = (k-j)(\Theta_1+\Theta_2)+\Theta_3 \\ \theta_2 = j\lambda' + (k-j)\lambda - k\gamma' & \Lambda' = j(\Theta_1+\Theta_2)+\Theta_4 \\ \theta_3 = \lambda & \Gamma = k\Theta_1 \\ \theta_4 = \lambda' & \Gamma' = k\Theta_2 \end{array} \tag{30}$$

Plugging in

$$\begin{aligned}\mathcal{H} &= -\frac{G^2 m_c^2 m^3}{2[(k-j)(\Theta_1+\Theta_2)+\Theta_3]^2} - \frac{G^2 m_c^2 m'^3}{2[j(\Theta_1+\Theta_2)+\Theta_4]^2} - \frac{G^2 m_c m m'^3}{[j(\Theta_1+\Theta_2)+\Theta_4]^2} f_d \left(\frac{2\Gamma}{\Lambda}\right)^{\frac{k}{2}} \cos\theta_1 - \text{S.C.} \\ &= -\frac{G^2 m_c^2 m^3}{2[(k-j)(\Theta_1+\Theta_2)+\Theta_3]^2} - \frac{G^2 m_c^2 m'^3}{2[j(\Theta_1+\Theta_2)+\Theta_4]^2} \\ &\quad - \frac{G^2 m_c m m'^3}{[j(\Theta_1+\Theta_2)+\Theta_4]^2} \frac{(2k\Theta_1)^{k/2}}{[(k-j)(\Theta_1+\Theta_2)+\Theta_3]^{k/2}} f_d \cos\theta_1 \\ &\quad - k\Theta_1\dot{\varpi}_{sec} + [\Theta_3 + (k-j)\Theta_1]\dot{\lambda}_{sec} - k\Theta_2\dot{\varpi}'_{sec} + (j\Theta_1+\Theta_4)\dot{\lambda}'_{sec}\end{aligned} \tag{31}$$

With approximations

$$\frac{1}{[\Theta_3+(k-j)(\Theta_1+\Theta_2)]^2} \approx \frac{1}{\Theta_3^2} - 2(k-j)\frac{\Theta_1+\Theta_2}{\Theta_3^2} + 3(k-j)^2\frac{(\Theta_1+\Theta_2)^2}{\Theta_3^4} + \ldots$$

$$\frac{1}{[\Theta_4+j(\Theta_1+\Theta_2)]^2} \approx \frac{1}{\Theta_4^2} - 2j\frac{\Theta_1+\Theta_2}{\Theta_4^3} + 3j^2\frac{(\Theta_1+\Theta_2)^2}{\Theta_4^4} + \ldots$$

$$\Theta_3 \approx \Lambda \quad \Theta_4 \approx \Lambda' \quad n^2 \approx \frac{Gm_c}{a^3} \quad n'^2 \approx \frac{Gm_c}{a'^3}$$

We get the general Hamiltonian

$$\begin{aligned}
\mathcal{H} = &-\frac{G^2m_c^2m^3(-2)(k-j)(\Theta_1+\Theta_2)}{2\Theta_3^2} + \frac{3G^2m_c^2m^3(k-j)^2(\Theta_1+\Theta_2)^2}{2\Theta_3^4} \\
&- \left\{ -\frac{G^2m_cm'^3 2j(\Theta_1+\Theta_2)}{2\Theta_4^3} + \frac{3G^2m_cm'^3j^2(\Theta_1+\Theta_2)^2}{2\Theta_4^4} \right\} \\
&- \frac{G^2m_cmm'^3}{[j(\Theta_1+\Theta_2)+\Theta_4]^2}\frac{(2k\Theta_1)^{k/2}}{[(k-j)(\Theta_1+\Theta_2)+\Theta_3]^{k/2}} f_d\cos\theta_1 \\
&- [k\dot{\varpi}_{sec} + (k-j)\dot{\lambda}_{sec} + j\dot{\lambda}'_{sec}]\Theta_1 \\
= &\ \alpha k\Theta_1 + \beta k^2\Theta_1^2 + \epsilon(2k\Theta_1)^{k/2}\cos k\theta_1 \\
= &\ \alpha\Gamma + \beta\Gamma^2 + \epsilon(2\Gamma)^{k/2}\cos k\theta_1
\end{aligned}$$

Where

$$\alpha = G^2m_c^2\left(\frac{m^3(k-j)}{\Lambda^2} + \frac{m'^3j}{\Lambda'^2}\right)\frac{1}{k} - [k\dot{\varpi}_{sec} + (k-j)\dot{\lambda}_{sec} + j\dot{\lambda}'_{sec}]\frac{1}{k}$$

$$\beta = \frac{3}{2k^2}\left(\frac{G^2m_cm'^3j^2}{\Theta_4^4} + \frac{G^2m_cm^3(k-j)^2}{\Theta_4^4}\right)$$

$$\epsilon = -\frac{G^2m_cmm'^3}{[j(\Theta_1+\Theta_2)+\Theta_4]^2}\frac{1}{[(k-j)(\Theta_1+\Theta_2)+\Theta_3]^{k/2}} f_d$$

$$\mathcal{H} = \alpha\Gamma + \beta\Gamma^2 + \epsilon(2\Gamma)^{k/2}\cos k\theta_1$$